SUPPLY CHAIN PLANNING MODELS WITH GENERAL BACKORDER PENALTIES, SUPPLY AND DEMAND UNCERTAINTY, AND QUANTITY DISCOUNTS

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SUPPLY CHAIN PLANNING MODELS WITH GENERAL BACKORDER PENALTIES, SUPPLY AND DEMAND UNCERTAINTY, AND QUANTITY DISCOUNTS

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To the most precious people in my life; my adorable parents, to the newest lovely member of our family; my nephew Hussein Megahed, and to the martyrs of the Egyptian revolution

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SUMMARY

In this thesis, we study three supply chain planning problems. The first two problems fall in the tactical planning level, while the third one falls in the strategic/tactical level. We present a direct application for the first two planning problems in the wind turbines industry. For the third problem, we show how it can be applied to supply chains in the food industry.

Many countries and localities have the explicitly stated goal of increasing the fraction of their electrical power that is generated by wind turbines. This has led to a rapid growth in the manufacturing and installation of wind turbines. The globally installed capacity for the manufacturing of different components of the wind turbine is nearly fully utilized. Because of the large penalties for missing delivery deadlines for wind turbines, the effective planning of its supply chain has a significant impact on the profitability of the turbine manufacturers. Motivated by the planning challenges faced by one of the world's largest manufacturers of wind turbines, we present a comprehensive tactical supply chain planning model for manufacturing of wind turbines in the first part of this thesis. The model is multi-period, multi-echelon, and multi-commodity. Furthermore, the model explicitly incorporates backorder penalties with a general cost structure, i.e., the cost structure does not have to be linear in function of the backorder delay. To the best of our knowledge, modeling-based supply chain planning has not been applied to wind turbines, nor has a model with all the above mentioned features been described in the literature. Based on real-world data, we present numerical results that

show the significant impact of the capability to model backorder penalties with general cost structures on the overall cost of supply chains for wind turbines.

With today's rapidly changing global market place, it is essential to model uncertainty in supply chain planning. In the second part of this thesis, we develop a two-stage stochastic programming model for the comprehensive tactical planning of supply chains under supply uncertainty. In the first stage, procurement decisions are made while in the second stage, production, inventory, and delivery decisions are made. The considered supply uncertainty combines supplier random yields and stochastic lead times, and is thus the most general form of such uncertainty to date. We apply our model to the same wind turbines supply chain. We illustrate theoretical and numerical results that show the impact of supplier uncertainty/unreliability on the optimal procurement decisions. We also quantify the value of modeling uncertainty versus deterministic planning.

Supplier selection with quantity discounts has been an active research problem in the operations research community. In this the last part of this thesis, we focus on a new quantity discounts scheme offered by suppliers in some industries. Suppliers are selected for a strategic planning period (e.g., 5 years). Fixed costs associated with suppliers' selection are paid. Orders are placed monthly from any of the chosen suppliers, but the quantity discounts are based on the aggregated annual order quantities. We incorporate all this in a multi-period multi-product multi-echelon supply chain planning problem and develop a mixed integer programming (MIP) model for it. Leading commercial MIP solvers take 40 minutes on average to get any feasible solution for realistic instances of our model. With the aim of getting high-quality feasible solutions quickly, we develop an

algorithm that constructs a good initial solution and three other iterative algorithms that improve this initial solution and are capable of getting very fast high quality primal solutions. Two of the latter three algorithms are based on MIP-based local search and the third algorithm incorporates a variable neighborhood Descent (VND) combining the first two. We present numerical results for a set of instances based on a real-world supply chain in the food industry and show the efficiency of our customized algorithms. The leading commercial solver CPLEX finds only a very few feasible solutions that have lower total costs than our initial solution within a three hours run time limit. All our iterative algorithms well outperform CPLEX. The VND algorithm has the best average performance. Its average relative gap to the best known feasible solution is within 1% in less than 40 minutes of computing time.

Chapter I

INTRODUCTION

A supply chain network is an integrated system that links a series of inter-connected business practices/processes that aid in getting a supply of raw materials, transforming them to end products, and distributing these end products to customers [1]. A supply chain is meant to facilitate the exchange of information along its different echelons (e.g., suppliers, transformation facilities, distribution centers, and customers). Its main objective is to increase profitability and enhance different operational activities.

With the increased complexity of structure and scale of today's supply chains, it becomes vital to develop analytical tools that help decision makers perform supply chain planning effectively. Depending on the time horizon, there are three levels of supply chain planning: strategic, tactical, and operational. Among others, Vidal and Goetschalckx [2], and Simchi-Levi et al. [3] mentioned the different planning decisions in each level. The strategic level deals with time horizons that are more than a year and have long-lasting impacts on the company. It entitles decisions like facility locations, numbers, and capacities, supplier selection, mode of transportation choice, and pricing. For the tactical level, the time ranges from a month/quarter to a year. Decisions here include demand allocation, inventory management, production/distribution coordination, production planning, and procurement. The operational level considers short-term decisions over hours, days, or at most weeks. Such decisions include production scheduling, truck loading, dispatching, and routing.

In this thesis, we study three supply chain planning problems. The first two problems fall in the tactical level category, while the third one considers a strategic/tactical problem. For all three problems, we consider a multi-periods, multi-products, multi-echelons supply chain with a multi-leveled bill of materials (BOM), inventory, manufacturing, and distribution considerations. We also include capacity restrictions at the manufacturing facilities, capacities at suppliers, and customer demand per period for each product. The first and third problems consider different aspects of deterministic supply chain planning, while the second problem deals with supply chain planning under uncertainty.

The rest of this chapter is outlined as follows: In Section 1.1, we give a brief problem definition for each of the three problems considered in this thesis and illustrate the research questions related to each. Then, we introduce some background for the technical methodologies used to solve each of our problems in Section 1.2. In Section 1.3, we state the thesis scientific contributions. Note that the detailed problem description and methodological development for each problem will be explained in its respective separate following chapters.

1.1 Problems Definition and Research Questions

Wind turbines have grown in the previous few years as an alternative source of green energy. The supply chain associated with the manufacturing and assembly of wind turbines is typically global and complex, and thus its effective supply chain management is necessary [4]. It is characterized with very expensive backorder penalties paid to customers whenever a delivery of a wind turbine is late. That penalty is a function of how long the delay is. However, it is not always a linear function. We present the first analytical model for the tactical planning of wind turbines supply chain and apply it to the supply chain of one of the world's biggest wind turbines manufacturers. Our model handles the aforesaid backorder penalties for the first time. Given this supply chain, we are faced with the following research questions:

- How can we determine the optimal procurement quantities, product flows, inventory levels, and manufacturing quantities across this supply chain?
- How do backorder penalties impact the optimal decisions?
- How can we include backorder penalties that are nonlinear functions of the backorder delay?
- How useful is it to be able to model such general backorder cost structure rather than just approximating it with a linear cost structure?

Uncertainty is one of the realistic and important features in today's supply chains, but it is very challenging to include in analytical models. In our second problem, we extend the first problem to include uncertainty. We focus on a new form of supplier uncertainty, which is not uncommon in practice. This supplier uncertainty is a combination of stochastic supplier lead times and random supply yield. The research questions one faces for this problem are:

- How can we model this problem and include that general case of supplier uncertainty?

- Do optimal procurement decisions and chosen suppliers differ when supplier uncertainty is modeled?
- What is the quantifiable improvement when uncertainty is modeled versus just using deterministic planning?

Supplier selection is another commonly studied issue in the literature of supply chain planning. In the third problem, we study a similar problem to the first one except that we include supplier selection with a new realistic quantity discount scheme, where the discount is based on the total aggregated annual order quantities. Suppliers are selected for a strategic period of time (typically 3 to 5 years). There is a fixed cost for each selected supplier. However, orders are placed in each time period (typically a month). We show an application of this scheme in a supply chains in the food industry. Given this definition, the research questions for this part are as follows:

- How can this problem be modeled?
- How time-efficient is it to get good feasible solutions quickly using leading commercial optimization solvers?
- Can we construct customized algorithms that generate high quality solutions quickly?
- How does using such algorithms compare to solving the model using the commercial solvers?

1.2 Technical Preliminaries

We introduce the basic concepts for the different technical methodologies used in this thesis; mixed integer programming (and linear programming), stochastic programming, local search, and variable neighborhood descent.

1.2.1 Mixed Integer Programming

A mixed integer linear programming (MIP) problem is given by:

min	$c^T x$		1.1
s.t.	$Ax \geq b$		1.2
	$x_j \in \mathbb{Z}_+$	$\forall j \in G$	1.3
	$x_j \geq 0$	$\forall j \in C$	1.4

Where x is a $n-$dimensional vector of decision variables, $\mathbb{Z}_+$ is the set of non-negative integers, and $G \subseteq \{1, \dots, n\}$ is the set of integer variables. These variables might be further restricted to be either zero or 1, in which case they are called binary variables. C is the set of continuous variables. The sets G and C partition the set of variables $\{1, \dots, n\}$. Let $\mathbb{Q}$ be the set of rational numbers, $c \in Q^m$ is the cost vector, $A \in Q^{mxn}$ is the constraint matrix, and $b \in Q^m$ is the right hand side vector for the constraints. Let X be the feasible region of the above problem, and $conv(X)$ denote the convex hull of X. In this case, any point $x \in X$ is an integer feasible solution of the MIP. If set $G = \emptyset$, then the above problem becomes a linear programming (LP) problem, which is much easier to solve than a MIP. Also, if we relax the integrality constraints (1.4), then the resulting problem is called the *LP relaxation of the MIP*. Most algorithms that are used to solve MIPs start with solving the LP relaxation.

MIPs are *NP-hard* optimization problems. Leading commercial solvers (e.g., CPLEX [5] and Gurobi [6]) provide multiple techniques to solve MIPs. Typically, they use a “branch-and-cut” procedure, which combines the methods of “branch-and-bound” and "cutting-planes". In branch-and-bound, some kind of smart enumeration of the complete solution space is done via pruning nodes that will not lead to any of the optimal solution(s). Branch-and-cut introduces valid inequalities that strengthen the LP relaxation by excluding parts of the solution space that do not contain any of the integer feasible solutions. For a review of the latter two methods and a comprehensive treatment of MIPs, we refer the reader to [7] and [8]. As for linear programming, we refer the reader to [9] and [10].

1.2.2 Stochastic Programming

Optimization under uncertainty has been an active research area for many years. There are multiple classifications for different stochastic optimization techniques/methods. We only use two-stage stochastic programming with recourse, and thus solely focus on this class of problems here. The dynamics of this problem is as follows: We make a decision x before observing the realization of a random event ξ. Then, we take a recourse action, say $y(x,\xi)$. Hence, x and y are the vectors of the first and second stage decision variables, respectively. Following [11], a two-stage stochastic linear program can then be expressed as follows:

$$\begin{aligned} \textbf{\textit{min}} \quad & c^T x + \mathbb{E}[Q(x,\xi(\omega))] && 1.5 \\ \textbf{\textit{s.t.}} \quad & Ax = b && 1.6 \\ & x \geq o && 1.7 \end{aligned}$$

Where $Q(x,\xi)$ is the optimal value of the following second stage problem:

$$\begin{aligned} \textbf{\textit{min}} \quad & q^t y && 1.8 \\ \textbf{\textit{s.t.}} \quad & Tx + Wy = h && 1.9 \\ & y \geq 0 && 1.10 \end{aligned}$$

The second stage problem depends on the data $\xi = (q,h,T,W)$, where some or all of the aforesaid parameters may have random values. The distribution of $\xi(\omega)$ is assumed to be known. Matrix T is called the technology matrix, and matrix W is called the recourse matrix. If the W matrix is not random, our problem is said to have a *fixed recourse.* If, for any decisions we take in the first stage, one can always construct a feasible solution for the second stage problem, the whole problem is said to have a *complete recourse.* The rest of the notation follows that of the MIP presented before.

In some cases, ξ might have a discrete (finite) distribution of K possible realizations, i.e., $\xi_k = (q_k, h_k, T_k), k = 1, \dots, K$, with corresponding probabilities p_k. We can then include all the possible realizations (sometimes called scenarios), and calculate $\mathbb{E}[Q(x,\xi)] = \sum_{k=1}^{K} p_k Q(x, \xi_k)$, where $Q(x, \xi_k) = \min\{q_k^T y_k : T_k x + W y_k = h_k, y_k \geq 0\}$. Hence, the problem (1.5-1.10) can be formulated as one large-scale linear program as follows:

$$\textbf{\textit{min}} \quad c^T x + \sum_{k=1}^{K} p_k q_k^T y_k \qquad 1.11$$

$$\textbf{\textit{s.t.}} \quad Ax = b \qquad 1.12$$

$$T_k x + W_k y_k = h_k, k = 1, \dots, K \qquad 1.13$$

$$x \geq 0 \qquad 1.14$$

$$y_k \geq 0, k = 1, \dots, K \qquad 1.15$$

Linear program (1.11-1.15) has a special block structure that makes it possible to be solved efficiently using different decomposition techniques. The most popular among such techniques is the so-called *L-shaped method* (see [12]).

First stage and/or second stage variables can be integers, by extending the above formulation to include integrality restrictions. In this case, the stochastic program is harder to solve (in particular when the second stage variables are integers). For a more in-depth analysis of the theory of stochastic programming, its solution algorithms, and extensions, we refer the reader to the texts of [13], [11], and [14].

1.2.3 Local Search

Because realistically-sized instances of MIPs are often very difficult to solve, development of fast heuristics has become a common research practice [15]. Local search algorithms are among the most popular examples of such heuristics. Beginning with an initial feasible solution $x^{(0)}$, a local search algorithm is an iterative procedure that searches the "neighborhood" of any given solution for an improved one. The procedure stops when there is no improved solution to the current solution. In this latter case, the algorithm is said to have reached a *local optimum* solution. Ghiani et al. [15] outlines a general local search algorithm as follows:

"*Step 0*. *Initialization*. Let $x^{(0)}$ be the initial feasible solution and let $N(x^{(h)})$ be its neighborhood. Set $h = 0$.

Step 1. Enumerate the feasible solutions belonging to $N(x^{(h)})$. Select the best solution $x^{(h+1)} \in N(x^{(h)})$.

Step 2. If the cost of $x^{(h+1)}$ is less than that of $x^{(h)}$, set $h = h + 1$ and go back to Step 1; Otherwise, STOP, $x^{(h)}$ is the best solution found."

1.2.4 Variable Neighborhood Descent

Variable neighborhood descent (VND) is a meta-heuristic that has been used successfully to solve hard optimization problems efficiently. Changing the neighborhood in a systematic way within a randomized local search algorithm is the main idea behind VND. See [16] and [17] for a detailed treatment of this method and an illustration of how it can be applied to different optimization problems. Algorithm 1 below is adopted from [16] and [17] that present the basic steps of VND.

Algorithm 1 Algorithmic Steps of the Basic VND Adopted from [16] and [17]

Initialization:
Select the set of neighborhood structures N_k; $k = 1, \dots, k_{max}$.
Find an initial solution x.
Repeat the following until no improvement is obtained:
$k \leftarrow 1$
while $k \leq k_{max}$ **do**
Find the best neighbor x' of $x(x' \in N_k(x))$.
if $x' \text{ is better than } x$ **then**
$x \leftarrow x'$
else
$k \leftarrow k + 1$
end if
end while

1.3 Thesis Contributions

This thesis presents various contributions. We state the contributions for each of our three studied problems as follows:

- For the first problem:
  - Developing a comprehensive tactical supply chain planning model for the wind turbines industry for the first time.
  - Modeling delay-dependent backorder costs of any functional form, and coming up with sufficient conditions or additional constraints to avoid the so called multi-hop backorders.
- For the second problem:
  - Developing a stochastic model for comprehensive tactical supply chain planning with a general form of supplier uncertainty.
  - Studying the effect of supply uncertainty on procurement decisions, both theoretically and empirically.
  - Applying our modeling approach to a real-world wind turbines supply chain.
- For the third problem:

- Defining and developing a modeling approach for a time-aggregated quantity discounts scheme.
- Developing different customized solution algorithms for the problem based on MIP-based local search and VND using novel neighborhoods.
- Applying our model to a realistic food supply chain.
- Illustrating the efficiency of using our customized algorithms in getting high quality solutions quickly compared to a leading commercial solver.

The rest of this thesis is outlined as follows: In chapters II, III, and IV, we present the three studied problems, review the literature related to each problem, illustrate a detailed problem definition and the used methodology to solve it, and show numerical results. In chapter V, we end with our conclusions and directions for future research.

Chapter II

TACTICAL PLANNING FOR A WIND TURBINES SUPPLY CHAIN CONSIDERING DELAY DEPENDENT BACKORDER PENALTIES WITH A GENERAL COST STRUCTURE

2.1 Introduction

Effective supply chain management is vital in the wind turbines industry [4]. Conversion of wind energy to electricity is the most rapidly growing renewable energy source in the world [18]. There has been a constant global growth in the installed capacity of wind turbines [19], with an average annual growth rate in the U.S. over the past five years of 33% [20]. In 2012, wind energy constituted 43% of all newly installed electricity generating capacities in the US [21]. Nevertheless, to the best of our knowledge, there has not been any application reported in the literature for supply chain models in that industry. This chapter presents a comprehensive supply chain planning model and case study for world's second biggest manufacturer of wind turbines [22].

Backorder penalties paid to customers for late deliveries can be critical for the economic success of a project in the wind turbines industry, especially with the ever growing demand and tight manufacturing capacities in this industry (see e.g. [19]). With these challenges in mind, it is no longer feasible to solely rely on ad hoc planning using spreadsheets as is the common practice. Merely devising a feasible supply chain plan became in itself very challenging and time consuming. A model-based supply chain planning tool combined with optimization thus represents a critical need.

Backorder penalties in our case study depend on the fulfillment delay, which is defined as the time lapse from the period the demand is backordered to the later period when the demand is fulfilled. Hsu and Lowe [23] used the term "period-pair-dependent" for this type of backorder cost function. We will refer to it as the "delay dependent" backorder. This backorder penalty function is not necessarily linear with respect to the backorder delay. To the best of our knowledge, prior results published in the literature only treat the case of linear cost structures (see e.g. [24]). More general cost structures were presented in [23], but for the economic lot sizing problem. We will discuss cost structures for delay dependent penalties in more details in Section 2.2 of this chapter.

Our supply chain model can handle backorder costs that have any cost structure in function of the backorder delay. Note that a piecewise convex or an exponential cost structure may be stipulated by the customers in order to penalize long fulfillment delays. These cost structures were observed in the case study that we present.

Therefore, the contributions of this chapter are twofold: First, we present a model and case study in a new tactical supply chain planning application (wind turbines). Second, we develop a modeling framework for general backorder delay dependent cost structures. We also show the impact of approximating, for instance, piecewise linear convex cost structures by linear functions on the costs incurred. In addition, we illustrate that adding backorder costs in general can have significant effects on the overall supply chain costs and on the supply chain planning decisions.

The remainder of this chapter is structured as follows: in Section 2.2, we review the literature on tactical supply chain planning and backorder costs. We then explain the real world supply chain case study that inspired this work, and develop our model in Section 2.3. In Section 2.4, we highlight and discuss some numerical results.

2.2 Literature Review

Depending on the scope of the planning decisions, the levels of planning for supply chains are typically divided into three classes: strategic, tactical and operational [2]. Numerous articles exist that focus on the strategic decisions such as facility locations and capacities [25]. Many of these articles include some tactical decisions as well. However, fewer articles focus solely on the tactical planning.

In their reviews, Vidal and Goetschalckx [2], Beamon [26], and Melo et al. [25] provided a summary of the different considerations that are usually included in supply chain planning models (sometimes referred to as production-distribution models). According to the authors, the main tactical considerations are: multi-period planning, multi-commodity, transportation/distribution, inventory, manufacturing issues (such as the bill of materials (BOM), production capacity at plants, and storage limitations), capacity of transportation channels, supplier selection, capacity at suppliers, distribution center capacities, and number of echelons in the supply chain. Costs typically included are the transportation cost, production cost, inventory holding cost, and backorder/backlog penalties.

Comprehensive supply chain planning models that do not include backorder costs have been proposed in [27-30], among others. Each of these papers includes production, inventory, transportation, and distribution for strategic/tactical design of multi-echelon multi-period multi-commodity supply chains. Comprehensive supply chain planning models in the literature that include backorder costs can be found in [24, 31, 32]. All of these latter models consider only linear backorder penalties. The recent model of Stadtler [24] is the only one that includes backorder cost structures that are a function of the backorder delay, but it treats only linear penalties.

The only models we could find in the literature that include nonlinear backorder costs are lot sizing/production related models (i.e., models that do not include distribution). We differentiate between two different categories here. One category includes backorder costs that do not depend on the delay. For those, the nonlinearity of the costs is a function of the backorder quantity. Thus, it is a different type of nonlinearity than the one we consider. Examples include the papers of Blackburn and Kunreuther [33] and Swoveland [34]. In the first paper, the authors developed a dynamic economic lot size model with a concave backlogging cost function. In the second one, the backorder cost function was piecewise concave.

The second category can be observed in the works of Hsu and Lowe [23], Hsu [35], and Bai et al. [36]. In Hsu and Lowe [23], the authors introduced what they called the pp-dependent inventory and backorder costs to the classical economic lot size models. Then, Hsu [35] extended one of his earlier works to include age-dependent inventory and backorder costs to a finite-horizon dynamic economic lot size model for perishable products. Note that age-dependent costs are different from pp-dependent ones (see [35] for a detailed explanation of the difference). The backorder cost structures for the studied problems in both papers have some restrictions with respect to the delay (see Table 1 in

[35] for a summary of all the assumptions/restrictions). In Bai et al. [36], the authors presented an economic lot-sizing problem with perishable inventory, where delay dependent backlogging was allowed. They had multiple restrictions on the backlogging cost. With respect to the delay, the marginal cost of having more unfulfilled demand of period i, when reaching period t, has to be no bigger than having a similar additional unfulfilled demand of period j, when reaching the same period t, where $i<j<t$. Also, the function has to be the so called economies of scale function (see their paper and its references for a detailed definition of that function).

There are some recent models that concentrated exclusively on tactical planning in the literature. Examples of the ones that presented applications in the energy sector are [37-40]. Gunnarsson and Rönnqvist [37] considered the integrated production and distribution planning for a pulp company, and included some tactical planning decisions such as transportation, inventory, and distribution. Ren and Gao [38] developed a mixed-integer programming model for the integrated plan and evaluation of distributed energy systems, and applied it for a test year. Waldemarsson et al. [39] considered the integrated planning of the supply chain at a multi-facility pulp company. Their model included purchasing, production, transportation, and inventory issues. Their planning horizon was one year, with monthly time periods. Zhang et al. [40] proposed a mixed integer programming model for the tactical planning of switchgrass-based bioethanol supply chains, and applied it to a case study in the state of North Dakota in the US.

Other recent tactical supply chain planning applications, which are not in the energy sector, can be found in [41-44]. Ahumada and Villalobos [41] developed an integrated tactical planning model for the production and distribution of fresh produce.

They reported that the model is used for making decisions for a large fresh produce grower in Northwestern Mexico. Beaudoin et al. [42] presented a model that aimed at supporting the tactical wood procurement decisions with multiple facilities. Ouhimmou [43] focused on the tactical planning of a section of a furniture supply chain.

All of the previous three tactical planning models did not include backorder cost considerations. Taşkin and Ünal [44] included backorder/backlog costs in their tactical model for float glass manufacturing. However, the costs were linear and independent of any backorder delay. As we mentioned earlier, to the best of our knowledge, none of the articles in the literature presented a tactical supply chain planning model in the wind turbines industry. Also, none included backorder cost functions that are delay dependent and that may have any functional form. Note that we restricted our review here to deterministic models and to supply chain planning (production-distribution) or lot sizing (production) models.

We present a new real world supply chain planning problem in the wind turbines industry, and develop a useful comprehensive tactical planning model for that problem. The model is multi-echelon, multi-period, multi-commodity, and handles transportation, inventory, distribution, production, BOM, resource capacity restrictions, and backorder aspects. Backorder delay penalties may have any functional form. We will begin with discussing the problem under study then present our model.

2.3 Problem Definition and Model Development

Each wind turbine consists of different main components. These components are the tower, nacelle, bearings, blades (a set of 3 blades), gearbox, generator, and rotor.

The European Wind Energy Association [4], and He and Chen [19] reported similar main parts. Figure 1 shows an illustration of these parts, among other components.

The company provides multiple wind turbine models. Our case study includes seven different models. Customers can order different quantities from any of these models. Each model has identical component structure but different brands/models for each component. The BOM for each turbine model is known. Some components are shipped directly from suppliers to customer sites. Others are assembled in the company's manufacturing facilities and shipped as subassemblies to customer sites where the final wind turbine is assembled. Note that in order to assemble any wind turbine, all of its components must be already on-site. Otherwise, parts that arrived early are stocked at specific inventory costs until the remaining components arrive.

If a turbine is assembled later than the period of its scheduled installation, delay dependent backorder penalties have to be paid to the customer. Shortages are not allowed. Backorder penalties are very strict and are stated in the contracts with the customers. Note that backorder costs in our case study only refer to the aforementioned penalties. Liberopoulos et al. [45] defined a variety of different stock-out cost quantifications, including one which they called the variable contractual penalties and which is exactly the one in our case study. In addition, we ignore any future indirect cost, such as the loss of customer goodwill, as indicated in [45].

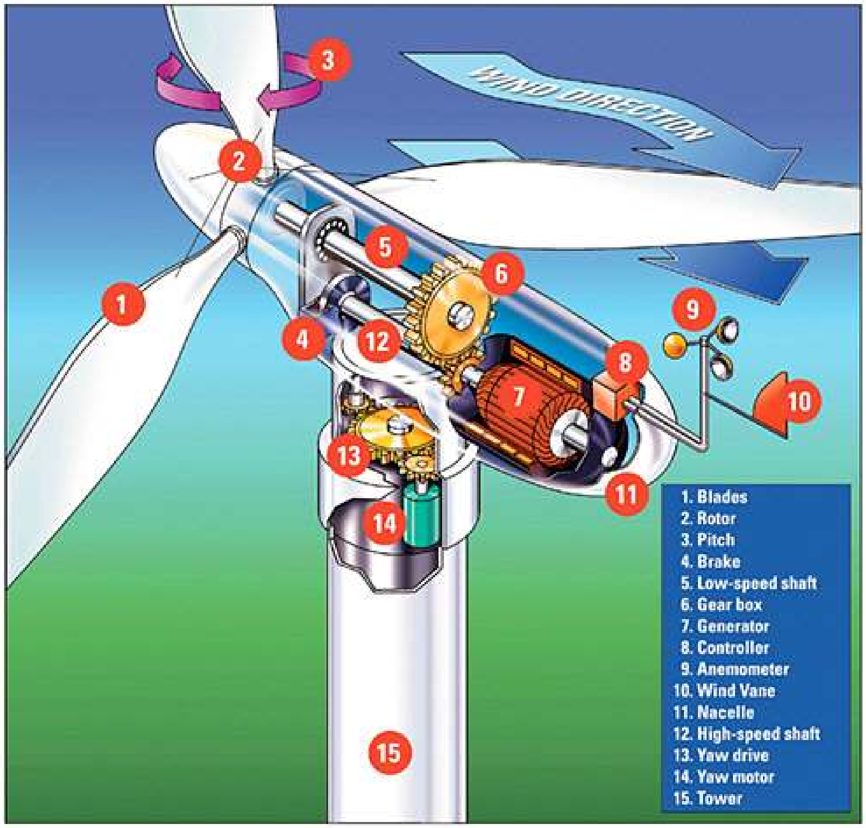

Figure 1 Components of a Wind Turbine, Reprinted with Permission from Parejo [46]

Potential suppliers are located all over the globe; therefore procurement decisions are significant with expensive transportation costs. In addition, supply capacities are usually tight (He and Chen [19] reported a similar result to this). Effective supply chain planning guarantees the production and timely delivery of goods for the company, while minimizing the total costs that include transportation, inventory, backorder, and other resource utilizations.

In our cast study, there are 42 different project sites dispersed over North America. The schedule to assemble and install a site-dependent number of wind turbines at each of the sites has been contractually agreed on. The tactical planning horizon is one year, composed of 52 periods (weeks). The planning can be directly incorporated in a rolling planning horizon framework (see Stadler [47] for an example and Sahin et al. [48] for a review).

There exist 25 different suppliers, some of which are facilities owned by the company itself. Note that suppliers are assumed to have been selected *a priori*. Supplier selection is typically a strategic decision and is not considered in our case study. The case study focusses on the tactical planning of procurement decisions. Thus, our formulation does not include any supplier fixed costs nor does it deal with any of the methods of supplier selection (see e.g. Ho et al. [49] for a review of such methods). There is a maximum capacity for assembling final products at each project site during any specific period. All the data is assumed to be known in advance. There is a holding cost per period per product for keeping one unit of each component or end product at each site. The company wants to determine the optimal flow quantities from suppliers to customer sites for each component at each period, the optimal assembled quantities, and the optimal stored components and end products, and backordered quantities. Because of the size of the problem instance and the various cost components, an optimization-based tool is required to determine these optimal quantities. Using spreadsheet-based manual planning, the company believed that they were off-schedule too frequently, and that they were incurring a lot of the expensive backorder penalties. They did not have any way to assess these impressions, and were not even able to check the feasibility of spreadsheet solutions for this large scale planning problem.

The supply chain considered in our model consists of three types of facilities: suppliers, transformation facilities, and customers. Suppliers provide products (either raw materials or semi-finished products) to transformation facilities where these products are processed (assembled or manufactured). Next, the processed materials are transported to either other transformation facilities for further processing or to customers to fulfill their demand. There is a multileveled recursive BOM for each product. The flow of products can only occur through predefined channels from suppliers to transformation facilities, between transformation facilities, or from transformation facilities to customers. The echelon structure of the supply chain can differ by product. Transformation facilities can be present at different stages of the supply chain for different products. Products can be kept in inventory at any transformation facility either before being processed, as raw material or semi-finished products, or after processing, as semi-finished or end products. Processing, inventory and throughput capacities are also considered in the model. In order to accommodate inventory of components and assembly capacity restrictions at the customer sites, a dummy transformation facility is collocated with each customer. Flow is only allowed from each dummy transformation facility to the customer at its site with a zero delivery cost.

The model is multi-product and dynamic, i.e., it has multiple periods. We next present the sets, parameters, variables, and formulation of our model.

2.3.1 Sets

S	Set of suppliers
P	Set of products
C	Set of customers
T	Set of periods
TF	Set of transformation facilities
R	Set of resources

FR, AR, IR, CR, SR	Sets of resources required for product flow (*FR*), assembly (*AR*) and product inventory (*IR*) in transformation facilities, those required for product transportation in the transportation channels (*CR*), and those required for production at suppliers (SR). These are the subsets of the set of resources *R*
$O = S \cup TF$	Set of Origin facilities, i.e., suppliers and transformation facilities
$D = TF \cup C$	Set of destination facilities, i.e., transformation facilities and customers
OD	Transportation channels, indexed by the combination of their origin and destination facilities

2.3.2 Parameters

$fcap_{irt}$	Aggregate capacity of throughput resource *r* at supplier *i* during period *t* for all products combined
$fcap_{jrt}$, $acap_{jrt}$, $icap_{jrt}$	Aggregate capacity of resource *r* at transformation facility *j* during period *t* for all products combined. The capacities are throughput, assembly and inventory capacities respectively
$fcap_{ipt}$	Capacity of throughput product *p* at supplier *i* during period *t*.
$fcap_{jpt}$, $acap_{jpt}$, $icap_{jpt}$	Capacity of product *p* at transformation facility *j* during period *t*. The capacities are throughput, assembly and inventory capacities respectively
$ccap_{ijrt}$	Aggregate capacity of transportation resource *r* in the transportation channel between facilities *i* and *j* during period *t* for all products combined transported
$ccap_{ijpt}$	Capacity of transporting product *p* in the transportation channel

between facilities i and j during period t

fc_{jpt}, ac_{jpt}, ic_{jpt}	Cost of flow (throughput), assembly, and holding (inventory) respectively, for a unit of product p at transportation facility j during period t (in case of inventory, the cost is for holding a unit from period t to the next period $t+1$
frc_{jrt}, arc_{jrt}, irc_{jrt}	unit resource cost of resource r for flow, assembly (production), and inventory respectively, at transformation facility of type j during period t
$fres_{jprt}$, $ares_{jprt}$, $ires_{jprt}$	Units of resource r consumed by one unit of product p shipped, assembled, and stored, respectively, at transformation facility j during period t.
$fres_{iprt}$	Units of resource r consumed by one unit of product p at supplier i during period t.
$cres_{ijprt}$	Units of resource r consumed by one unit of product p transported in the transportation channel between facilities i and j during period t. This and the previous resource parameters allow the model to incorporate resource consumption rates that vary by period, e.g., to approximate learning curves.
crc_{ijrt}	Cost of one unit of resource r in the transportation channel between facilities i and j during period t
cc_{ijpt}	Unit cost through the transportation channel from facility i to facility j for a unit of product p during period t
dem_{kpt}	Demand for product p at customer k during period t

pc_{ipt} Purchase cost for a unit of product p from supplier i during period t

bc_{kptu} Delay cost, i.e., delay penalty or backorder cost, for delivering one unit of product p during period t to satisfy demand during period u at customer k

$1\,bom_{jpvt}$ Number of units of component p required to assemble one unit of assembly v during period t in transformation facility j where component p is an element of the single level bill of material of product v

$init\_inv_{jp}$ Initial inventory of product p at transformation facility j

2.3.3 Decision Variables

pq_{ipt} Amount purchased from supplier i of product p during period t

x_{ijpt} Amount of product p transported from i to j during time period t, *where* $(i, j) \in OD$

ifq_{jpt}, ofq_{jpt} Amount of product p respectively transported into and out of transformation facility of type j during time period t

iq_{jpt} Amount of product p stored (carried as inventory to the next period) at transformation facility of type j from time period t to time period $t+1$

bq_{kptu} Backorder quantity of product p delivered to customer k during period t that is used to satisfy the demand of this customer for this product during time period u, where u is smaller than t

aq_{jpt} Amount of product p assembled, i.e., manufactured or produced, at transformation facility j during time period t

cq_{jpvt} Amount of component product p used in assembly/manufacturing of product v at transformation facility j during time period t

dq_{kpt} Amount of product p delivered to customer k during period t to satisfy the demand during this period and possible backordered quantities of prior periods.

2.3.4 Model Formulation

The complete tactical supply chain model is given next. The model can be further condensed by directly substituting variables, but it is given below in its more expanded form to clearer show its structure. Modern linear programming solvers will make the substitutions in their pre-solve phase, so this more expansive version does not increase solution time significantly.

$$
\begin{aligned}
Min \quad & \sum_{i \in S} \sum_{p \in P} \sum_{t \in T} pc_{ipt}.pq_{ipt} + \\
& \sum_{(i,j) \in OD} \sum_{p \in P} \sum_{t \in T} cc_{ijpt}.x_{ijpt} + \sum_{(i,j) \in OD} \sum_{p \in P} \sum_{t \in T} crc_{ijrt}.cres_{ijprt}.x_{ijpt} + \\
& \sum_{j \in TF} \sum_{p \in P} \sum_{t \in T} fc_{jpt}.ofq_{jpt} + \sum_{j \in TF} \sum_{p \in P} \sum_{r \in TR} \sum_{t \in T} frc_{jrt}.fres_{jprt}.ofq_{jpt} + \\
& \sum_{j \in TF} \sum_{p \in P} \sum_{t \in T} ac_{jpt}.aq_{jpt} + \sum_{j \in TF} \sum_{p \in P} \sum_{r \in AR} \sum_{t \in T} arc_{jrt}.ares_{jprt}.aq_{jpt} + \\
& \sum_{j \in TF} \sum_{p \in P} \sum_{t \in T} ic_{jpt}.iq_{jpt} + \sum_{j \in TF} \sum_{p \in P} \sum_{r \in IR} \sum_{t \in T} irc_{jrt}.ires_{jprt}.iq_{jpt} + \\
& \sum_{k \in C} \sum_{p \in P} \sum_{t \in T \backslash \{1\}} \sum_{u \in T: u < t} bc_{kptu}.bq_{kptu}
\end{aligned} \qquad 2.1
$$

$$
s.t. \quad \sum_{p \in P} fres_{iprt}.pq_{ipt} \le fcap_{irt} \qquad \forall i \in S, \forall t \in T, \forall r \in SR \qquad 2.2
$$

$$
pq_{ipt} \le fcap_{ipt} \qquad \forall i \in S, \forall p \in P, \forall t \in T \qquad 2.3
$$

$$
pq_{ipt} = \sum_{j \in TF} x_{ijpt} \qquad \forall i \in S, \forall p \in P, \forall t \in T \qquad 2.4
$$

$$\sum_{i \in O} x_{ijpt} = ifq_{jpt} \qquad \forall j \in TF, \forall p \in P, \forall t \in T \qquad 2.5$$

$$\begin{aligned} & ifq_{jpt} + aq_{jpt} + init\_inv_{jp} - iq_{jpt} \\ & - \sum_{v \in P} cq_{jpvt} - ofq_{jpt} = 0 \end{aligned} \qquad \forall j \in TF, \forall p \in P, t = 1 \qquad 2.6$$

$$\begin{aligned} & ifq_{jpt} + aq_{jpt} + iq_{jpt-1} - iq_{jpt} \\ & - \sum_{v \in P} cq_{jpvt} - ofq_{jpt} = 0 \end{aligned} \qquad \forall j \in TF, \forall p \in P, \ \forall t \in T \setminus \{1\} \qquad 2.7$$

$$\sum_{p \in P} cres_{ijprt} . x_{ijpt} \le ccap_{ijrt} \qquad \forall (i, j) \in OD, \forall t \in T, \forall r \in CR \qquad 2.8$$

$$x_{ijpt} \le ccap_{ijpt} \qquad \forall (i, j) \in OD, \forall p \in P, \forall t \in T \qquad 2.9$$

$$\sum_{j \in D} x_{ijpt} = ofq_{ipt} \qquad \forall i \in TF, \forall p \in P, t \in T \qquad 2.10$$

$$cq_{jpvt} = 1\, bom_{jpvt} . aq_{jvt} \qquad \forall j \in TF, \forall p \in P, \forall v \in P, \forall t \in T \qquad 2.11$$

$$\sum_{p \in P} ares_{jprt} . aq_{jpt} \le acap_{jrt} \qquad \forall j \in TF, \forall r \in AR, \forall t \in T \qquad 2.12$$

$$aq_{jpt} \le acap_{jpt} \qquad \forall j \in TF, \forall p \in P, \forall t \in T \qquad 2.13$$

$$\sum_{p \in P} fres_{jprt} . ofq_{jpt} \le fcap_{jrt} \qquad \forall j \in TF, \forall r \in FR, \ \forall t \in T \qquad 2.14$$

$$ofq_{jpt} \le fcap_{jpt} \qquad \forall j \in TF, \forall p \in P, \forall t \in T \qquad 2.15$$

$$\sum_{p \in P} ires_{jprt} . iq_{jpt} \le icap_{jrt} \qquad \forall j \in TF, \forall r \in IR, \forall t \in T \qquad 2.16$$

$$iq_{jpt} \le icap_{jpt} \qquad \forall j \in TF, \forall p \in P, \forall t \in T \qquad 2.17$$

$$\sum_{j \in TF} x_{jkpt} = dq_{kpt} \qquad \forall k \in C, \forall p \in P, \forall t \in T \qquad 2.18$$

$$dq_{kpt} + \sum_{t \in T : t < u} bq_{kput} = \sum_{u \in T : u < t} bq_{kptu} + dem_{kpt} \qquad \forall k \in C, \forall p \in P, \forall t \in T \qquad 2.19$$

$$pq_{ipt} \geq 0 \qquad \forall i \in S, \forall p \in P, \forall t \in T \qquad 2.20$$

$$x_{ijpt} \geq 0 \qquad \forall (i,j) \in OD, \forall p \in P, \forall t \in T \qquad 2.21$$

$$ifq_{jpt}, ofq_{jpt}, iq_{ipt}, aq_{jpt} \geq 0 \qquad \forall j \in TF, \forall p \in P, \forall t \in T \qquad 2.22$$

$$bq_{kptu} \geq 0 \qquad \forall k \in C, \forall p \in P, \forall t \in T, \forall u \in T \qquad 2.23$$

$$cq_{jpvt} \geq 0 \qquad \forall j \in TF, \forall p \in P, \forall v \in P, \forall t \in T \qquad 2.24$$

$$dq_{kpt} \geq 0 \qquad \forall k \in C, \forall p \in P, \forall t \in T \qquad 2.25$$

$$\sum_{u \in T: u<t} bq_{kptu} \leq dq_{kpt} \qquad \forall k \in C, \forall p \in P, \forall t \in T \setminus \{1\} \qquad 2.26$$

The objective function 2.1 computes the total cost as the sum of the products of the individual unit cost rates multiplied by the corresponding quantities. Costs included are purchasing, transportation, inventory holding, assembly, backorder, and different resource utilization costs.

The model contains four types of constraints: supply capacity, production or assembly capacity, conservation of flow at the transformation facilities, and demand satisfaction. Typically capacity limitations at suppliers are either for individual products or for all products combined. The former case is modeled in constraint 2.3, while the latter one is modeled in constraint 2.2. The model allows both simultaneously but usually only one of them is relevant in a particular instance. The equivalent is true for assembly capacities at transformation facilities modeled by constraint 2.12, which models the joint capacity, and/or constraint 2.13, which models the capacity for an individual product. The same applies to the transformation facilities throughput capacity through constraints 2.14 and 2.15, and inventory capacity at these facilities in constraints 2.16 and 2.17. Similarly, it applies to flow capacities at different transportation channels through constraints 2.8 and 2.9.

The conservation of flow constraint at any transformation facility for a product in a certain period has six flows. The three input flows are transportation receipts, inventory held from the previous period, and production during the period. The three output flows are transportation shipments, inventory held to the next period, and consumption of the product during the period when it is used as a component in the production process of another product. This general form, which covers transformation-space-time at the transformation facilities, is used in our model in constraints 2.6 and 2.7. Figure 2 shows these conservations.

Two variants of the conservation flow constraint (constraints 2.6 and 2.7 respectively) need to be created since the equation is different for the first period compared to all other periods of the planning horizon. During the first period there is only the initial inventory which is a parameter, while during all other periods, the inventory held from the previous period is a decision variable.

Constraints 2.4 and 2.18 ensure that all purchased products get transported from the suppliers and all produced finished goods get delivered to the customers, respectively. Constraints 2.5 and 2.10 relate the product flow to the input and output quantities at transformation facilities, respectively. Constraint 2.11 is the BOM constraint. It guarantees that the correct amounts of components are consumed in order to assemble finished/semi-finished goods. Constraint 2.19 ensures that the goods delivered to a customer and backorders from future periods are allocated to satisfy either the demand of that period or backorders in previous periods.

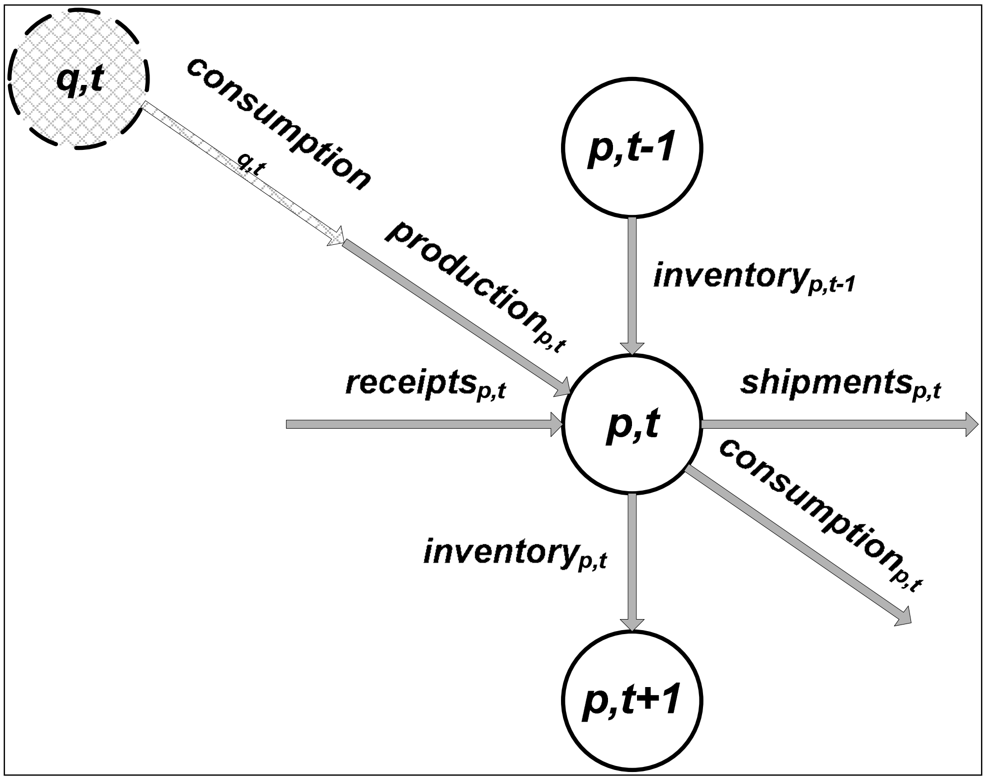

Figure 2 Transformation-Space-Time Conservation of flow

The conservation of flow for the customer backorders for a single product at a single customer is illustrated in Figure 3. The "deliveries" are goods delivered by the supply chain to this customer. Backorders satisfy demand from a later period to an earlier one in the planning horizon.

Note that in model (2.1-2.25), the backorder cost is defined in a way that is explicitly dependent on the backorder delay. The above model may yield infeasible solutions because of what we call the multi-hop phenomenon. Multi-hop is said to occur when backorder flows at a certain period satisfying the demand at an earlier period get transferred through one or more intermediate periods. See Figure 4 for an example, in which we assume that an optimal solution is generated in which quantities delivered at period *t* would be backordered for period *u*. Therefore, a backorder cost from *t* to *u* will be the true cost. However, the model might choose to do three backorders (from *t* to *f1*, *f1* to *f2*, and *f2* to *u*) even though no deliveries are made in these intermediate periods; because the sum of the three corresponding backorder costs is cheaper than the single backorder cost from *t* to *u*. Thus, the model will be calculating infeasible flows and a corresponding incorrect cost.

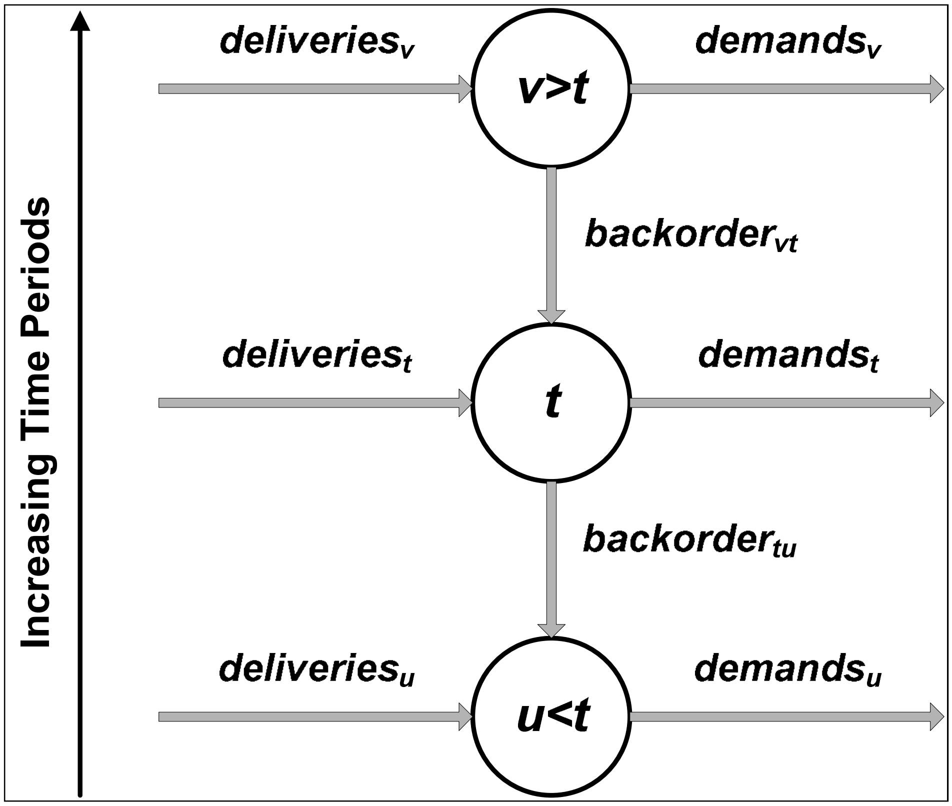

Figure 3 Conservation of Flow for Customer Backorders

To avoid this infeasible solution, we use constraint 2.26. This constraint guarantees that any backorder quantity from a period to an earlier period has to be part of the delivered quantities to the later period. There may be a large number of constraints of this constraint type, which may make the solution of the model more computationally demanding. We now present the following theorem:

Theorem 1:

If the backorder cost as a function in the backorder delay, say $f(\Delta d)$, is strictly subadditive, then constraint 2.26 need not be included in the model.

Proof:

From the definition of subadditivity, it will always be cheaper to avoid multi-hops and have a direct backorder from a later to an earlier period. Thus, no multi-hop solution will ever be part of an optimal solution. This holds if the unit cost is strictly decreasing, i.e., $f(\gamma.\Delta d) > \gamma.f(\Delta d), 0 < \gamma < 1$. ■

Note that concavity here is not required, but concavity implies subadditivity, and is thus sufficient to not include constraint 2.26. Another way to indicate this, using the same notation of the model, is equation 2.27.

$$bc_{kptu} < bc_{kptf} + bc_{kpfu} \qquad \forall k \in C, p \in P, t \in T, u \in T, f \in T : u < f < t \qquad 2.27$$

Condition 2.27 needs to be checked before the instance model is generated. This check can be executed in $O(|C| \text{ X } |P| \text{ X } |T|^3)$ time, where $|C|$, $|P|$, and $|T|$ are the cardinalities of sets C, P, and T, respectively. This check is not computationally demanding for real world instances. If the condition is satisfied, then constraint 2.26 should not be included in the model. Therefore, the following theorem has been proved:

Theorem 2:

Any delay dependent backorder cost structure can be included in our modeling framework. Strictly subadditive structures do not require the additional constraint 2.26 to be included in the model, while other structures do.

The following corollaries identify some backorder cost structures that satisfy the aforementioned condition 2.27. The proofs are provided in appendices A and B. Other delay dependent backorder cost structures may or may not satisfy the condition.

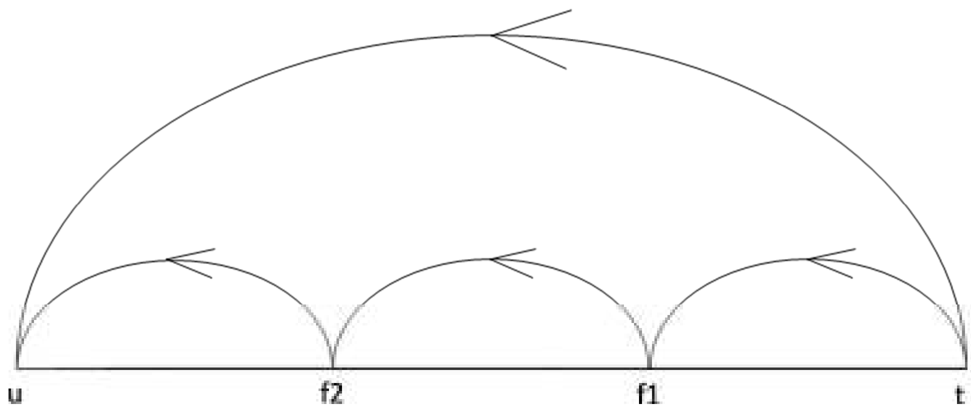

Figure 4 Illustration of the Multi-Hop Problem

Corollary 1:

A linear backorder cost structure with a positive intercept always satisfies condition 2.27.

Corollary 2:

A piecewise linear concave backorder cost structure, with a positive intercept of the first interval, always satisfies condition 2.27.

Lastly, constraints 2.20 through 2.25 are the non-negativity constraints. In the next Section, we present our numerical results and case study.

2.4 Numerical Results, Discussion, and Case Study

We apply our model to the real-world industrial case study described above. We also investigate the impact of different backorder cost structures on its performance, as well as the effect of backorder costs. Then, we show that being able to model a nonlinear delay dependent backorder cost structure can be significant. For each of these results, we illustrate how they can be useful for the real-world applications.

The model is coded in GLPK® [50], which is an open source optimization language and solved with glpsol® [50], under Windows XP®. We developed a database schema in Microsoft Access® that contains data fields for all the model parameters and variables in different tables, and read the data from it. The model is generated and solved, and the results are stored in the database. The machine used in the experiment has an Intel® core 2 duo T7200 with 2.00 GHZ for each processor and 3 G.B. of RAM. The instance of our case study takes 10 minutes for model generation including data retrieval and gets solved to optimality in 2 minutes. The model generated optimal solutions in a fraction of the time previously required to generate a solution using manual or spreadsheet-based planning.

Note that the supply chain analysts were never able to generate the optimal solutions manually. They could not even check the feasibility of any instance of the model. This is all treated when our model is used, which demonstrates its value for real-world applications. In addition, our model can be easily used by users that might not have a formal knowledge or training in operations research. Furthermore, using our model, they can quickly experiment changing the value of some of the model parameters and solving the resulting instances. This was very time-inefficient using manual or spreadsheet planning.

The case study is characterized by strongly binding supplier capacities and significant backorder penalties. The cost of our optimal solution is significantly lower than the cost of previous solutions. The model computes all the intricate tradeoffs between purchasing, transportation, and backorder decisions in a capacitated supply chain. The resulting flows may be counterintuitive at first.

2.4.1 Importance of Including Backorder Costs

In order to illustrate the importance of adding the backorder cost explicitly in our model, we solve two variants of the model for every set of input parameters. The first variant includes the backorder costs in the objective function as described above. The second variant does not include backorder cost in the optimization objective, but computes the optimal solution and then adds the backorder costs corresponding to the resulting optimal backorder flows. This calculated total cost is then compared with the objective function value of the first variant. We solved both variants for different backorder cost structures but restrict the comparison to linear cost structures.

Results show that there is a decrease in the total cost of 15% and up to 85% when the backorder cost is explicitly included in the objective function. The reduction is larger for steeper slopes of the backorder penalty function, most of which are realistic in the case study. Table 1 illustrates some of these reductions. This demonstrates the value of including the backorder penalties explicitly in the model for this particular case study.

Table 1 Effect of Adding Backorder Penalties to the Objective Function of Our Model

Slope (X1,000)	***% difference between overall cost without backorder penalty included and overall cost with backorder penalty included***
1	15%
10	32%
25	50%
50	66%
100	79%
150	85%

There are two applications of these results in the real-world application of the wind turbines. First, one can easily experiment the effect of different backorder costs on the overall supply chain cost using our model. Second, knowing how impactful the backorder penalties are on the overall cost, the company could focus on negotiating the contracts with customers. That is, they can try to include backorder cost structures and values that would decrease the overall supply chain cost. During the negotiation stage, they can always use our model to economically evaluate any negotiated values before signing the contracts.

2.4.2 Impact of Different Backorder Cost Structures

We next study the effect of different linear backorder cost structures on the overall cost and flows of our case study. Figure 5 shows the total objective function value in function of the slope of the backorder cost. The total objective function increases until the slope reaches a limit of 175,000, after which it remains constant. Figures 6 and 7 show the changes in the transportation cost, and the backorder and inventory costs in function of the cost slopes, respectively. The backorder cost first increases until a slope of 10,000, then strongly decreases until it reaches zero at a slope of 175,000 and finally remains zero for steeper slopes. The total cost behaves nearly identical to the transportation cost since the transportation costs makes up about 90% of the total cost.

The changes in the total cost are caused by the interaction of the backorder costs and the supplier capacities. For this particular case, the supplier capacity is strongly binding on the optimal solution. The optimal solution tends to use the closer and cheaper suppliers as much as possible by using backorders and inventories. When the backorder cost slope increases, the model makes a tradeoff between increasing backorder costs and sourcing from more distanced and/or more expensive suppliers. When the slope of backorder costs reaches 175,000, it is no longer cost effective to use any backordering and the model uses more expensive suppliers for all the unmet demand after consuming the capacity of nearby suppliers. Suppliers are in general nearly fully utilized in earlier periods of the planning horizon and as a consequence it is nearly impossible to build up inventory in the early periods. This makes the inventory cost almost constant in function of the slope of the backorder penalties.

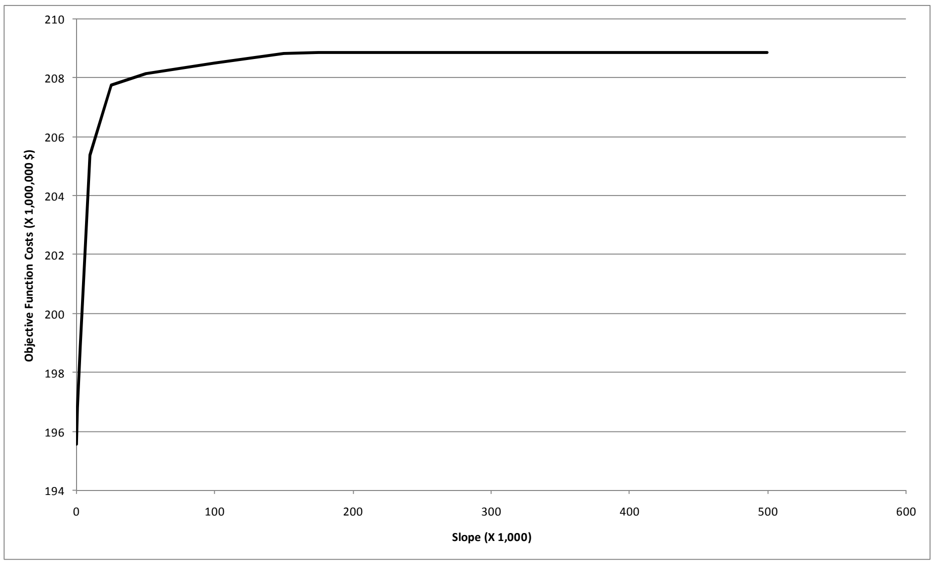

Figure 5 Objective Function Values for Different Slopes of the Linear Backorder Cost

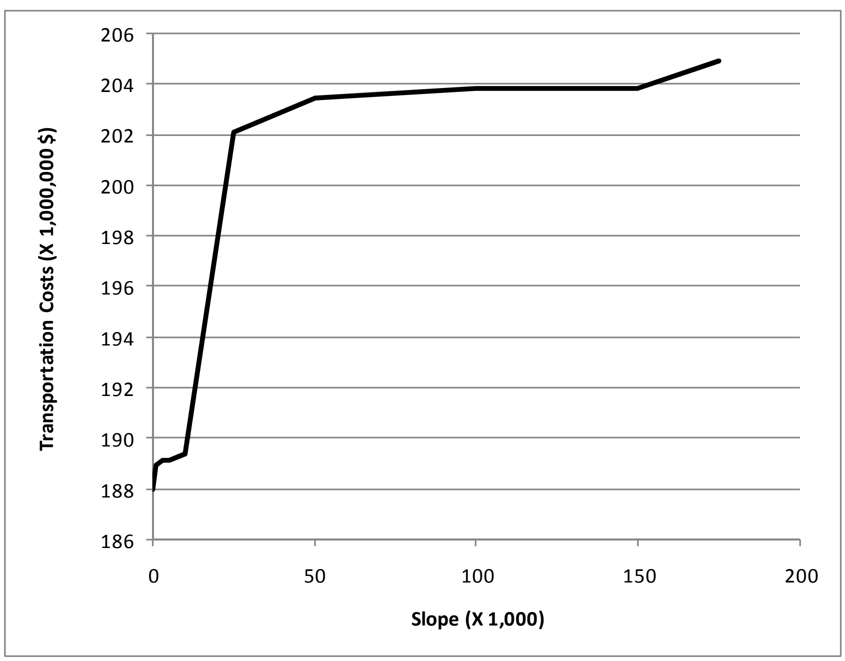

Figure 6 Transportation Cost for Different Slopes of the Linear Backorder Cost

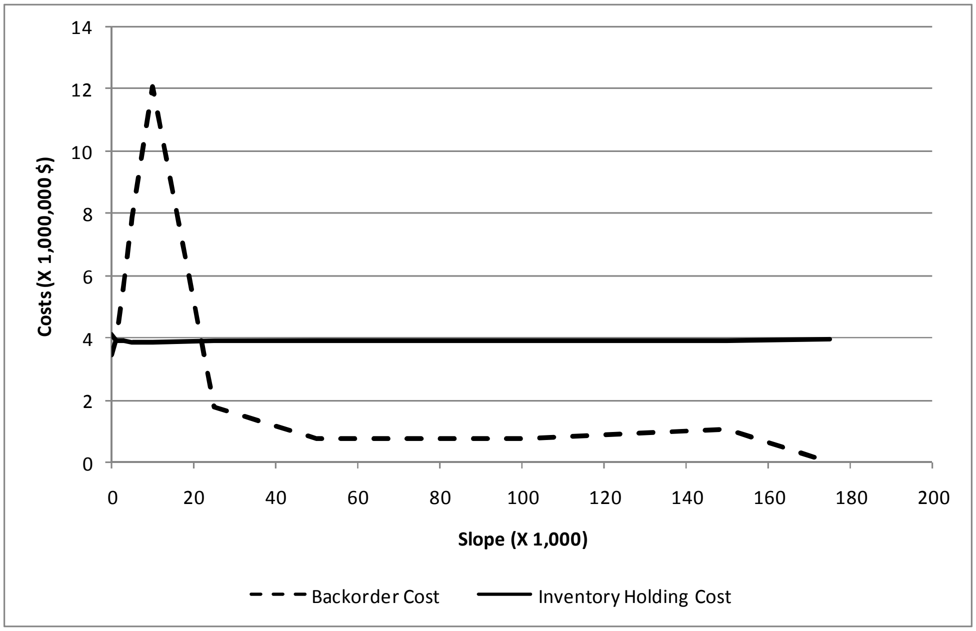

Figure 7 Inventory and Backorder Costs for Different Slopes of the Linear Backorder Cost

In order to verify the above explanation of the cost behavior, two additional sets of experiments were performed with additional supplier capacities. In the first experiment, we added 20% capacity to the original capacity of each supplier, while in the second one, we eliminated the supplier capacity constraints altogether. The results of each of these two cases are consistent with the results of the original case, but the overall objective function values are lower. For the case where there was no supplier capacity constraint, no backorder or inventory is needed. The cheapest suppliers are always selected. This results in a significant decrease of the overall cost. Figure 8 shows a comparison between the objective function values for the three cases.

These results illustrate the tight supply capacity in our wind turbine supply chain case study, as indicated earlier in this chapter as well as by other researchers. It also shows that it is very difficult to predict which combination of inventory, transportation, and backorder flows will constitute the optimal solution, especially for instances of real world size. Only a comprehensive planning model can find a feasible solution that also achieves the best tradeoffs. Manual solutions or sub optimization may lead to unnecessary delivery delays with their corresponding increasing backorder penalties.

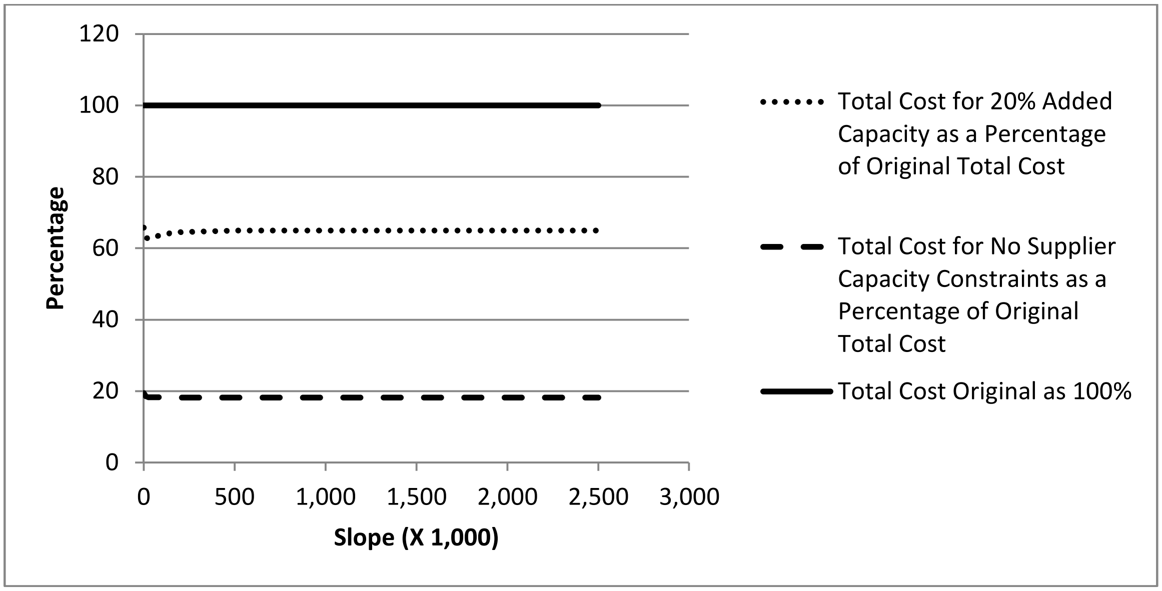

Figure 8 Total Costs for the 20% Added Supplier Capacity Case, and the No Supplier Capacity Case, as Percentages of the Original Case

This result entitles that the company should try to find more potential suppliers or see if the currently selected suppliers can increase their capacities. Whenever new suppliers are selected, they can be straightforwardly included in our model and the model can then be solved again to evaluate the new situation. This, again, is another illustration of how our model can be used in the real-world applications.

2.4.3 Significance of Modeling Some General Backorder Cost Structures

We illustrate how it can be significant to model general delay dependent backorder cost structure such as a three pieces convex cost structure. This cost structure is not uncommon for a real world case study.

We solve three variants of the model. The first one includes the piecewise linear convex cost structure with three line segments, where the first slope is four times the second slope, and the third slope is nine times the second one. The cutoffs between the first and second pieces is at a period difference of four, while that between the second and third pieces is at a period difference of 16. Note that constraint (2.26) needs to be included for this variant. The second variant is solving the model with a linear approximation of that cost structure, where the slope of the first piece is assumed to be the only slope. Then, we subtract the total optimal resulting backorder costs and add the backorder costs corresponding to the actual three piecewise convex cost function multiplied by the corresponding optimal backorder quantities of this variant. The last variant is similar to the second one except that we use a linear regression line, that passes through the origin, as an approximation of the three piecewise convex structure. We chose these two penalty approximations since they could be used within any of the models in the literature that handle only linear cost structures. We vary the cost slope of the first line segment and report the percentage differences in the increase of costs for each of the two approximations compared to the first variant. Note that the total cost (objective function value) of the first variant cannot be more than that of the second or the third variants; since the real backorder cost structure is included explicitly only in that first variant.

Table 2 summarizes the results. One can see that the percentage differences can reach up to over 6% of the total supply chain costs at realistic cost slopes. These percentage differences correspond to millions of dollars.

The previous results emphasize that our model should be used instead of previous models in the literature for supply chain planning with backorder cost structures that are not linear in the backorder delay. This is another useful application of our model in the real-world applications.

Table 2 Effect of Approximating a Three Piecewise Linear Convex delay dependent Backorder Cost Function

Slope (X1,000)	***% increase in cost for the approximation in the second variant***	***% increase in cost for the approximation in the third variant***
2	6.40	152.21
4	5.83	105.16
8	4.35	125.30
10	3.64	141.12
32	0.95	0.00
64	0.44	0.00
128	0.16	0.00

Chapter III

TACTICAL SUPPLY CHAIN PLANNING UNDER UNCERTAINTY WITH AN APPLICATION IN THE WIND TURBINES INDUSTRY

3.1 Introduction

The problem of supply chain planning became even more vital to the business practices of most manufacturing and service organizations because of the increasing competitive pressures and rapid advances in information technology in today's global marketplace [51]. With the changing market conditions and increasing customer expectations in the highly volatile business environment, the impact of uncertainties on planning in supply chains needs to be considered [52]. The treatment of uncertainty is one of the most challenging but important problems in supply chain management [53]. Multiple sources of uncertainty arise in supply chains, such as demand and supply uncertainties [54]. According to [55], stochastic programming is the most popular methodology for supply chain optimization under uncertainty. Stochastic programming (see [13], [11], and [14]), also referred to as optimization under uncertainty or planning under uncertainty, has witnessed significant progress in its methodology and practice in the past couple of decades [56].

In the previous chapter of this thesis, we introduced a deterministic approach for the planning of the supply chains of wind turbines. However, uncertainty and especially supply uncertainty is an important factor in this industry since it is strongly capacity constrained. Thus, an approach for solving this planning problem that explicitly incorporates such uncertainty is needed.

In this chapter, we present a two-stage stochastic programming model for comprehensive tactical supply chain planning under the most general form of suppliers' uncertainty to date. This uncertainty is a combination of supplier random yield and stochastic lead times. We present a model for the comprehensive planning of this general supplier uncertainty in Section 3.3 below. The problem definition and developed model were inspired by a real-world application in the wind turbines industry. We give an overview of this application, and show how our model can be used for the tactical supply chain planning problem of the world's second biggest manufacturer of wind turbines [22]. This work is an extension to the previous deterministic model in the previous chapter. We investigate the effect of the aforementioned uncertainty on optimal procurement decisions and show how the optimal choice of suppliers depends on the unreliability/uncertainty of these suppliers. Finally, we establish the value of using a stochastic model versus deterministic planning.

The contributions of this chapter are therefore threefold: First, a novel quantitative model for comprehensive tactical supply chain planning under the most general form of supplier uncertainty is developed. Second, the model is applied to the real-world application of the wind turbines manufacturing. Third, we present a theoretical and computational analysis to answer the research questions stated above.

The remainder of the chapter is structured as follows: in Section 3.2, we review the relevant literature, and then explain the problem under study along with the development of the two-stage stochastic programming model in Section 3.3. In Section 3.4, we describe the real world application of our model in the wind turbines industry and present numerical and theoretical.

3.2 Literature Review

Supply chain planning under uncertainty has recently been a very active research area. Peidro et al. [54] classified the literature on this topic according to three taxonomies: source of uncertainty, problem type, and modeling approach. Their sources of uncertainty included demand, process/manufacturing, and supply.

As for the modeling approach, they included four modeling approaches: analytical models, models based on artificial intelligence, simulation models, and some hybrid modeling. Our work uses stochastic programming and thus fits in the category of analytical models. Other comprehensive review articles on supply chains under uncertainty are the works [57], [58], [59], [55], [60], and [61].

We now focus our review on quantitative supply chain planning under uncertainty. Demand uncertainty is the most extensively studied source of uncertainty in supply chain planning [51]. Zhang and Saboonchi [62] mentioned that two-stage stochastic programming is widely used in the literature of supply chain planning under demand uncertainty. They developed a two-stage stochastic programming model that acts as a decision support tool for some logistical decisions in global supply chains under demand uncertainty.

In their successive papers, Tomasgard and Høeg [63], Schütz et al. [64], and Schütz et al. [65] presented a supply chain design problem for a Norwegian company in the meat industry. They formulated the problem as a two-stage stochastic program that incorporated the uncertainty in the demand. Kanyalkar and Adil [66] developed a robust optimization model for integrated aggregate planning of a multi-site procurement-production-distribution system, motivated by a real-world case study of a consumer goods company in India. Saboonchi and Zhang [67] considered the design of multi-stage global supply chains with stochastic demand. They used a two-stage stochastic programming model to aid with the tactical decisions of the selection of international outsourcing partners, transportation modes, and the capacity of facilities. Other examples for modeling supply chain planning problems under demand uncertainty using stochastic programming include [68], [69], [70], [52], and [51].

In this work we focus on supply uncertainty, since it is the main source of uncertainty in the application that inspired it. However, our model can easily address many other sources of uncertainty such as demand and capacity as we will show in Section 3.3. In our review of the literature, we observed that some authors consider the uncertainty in supply costs and capacities as "supply uncertainty", while others consider random supply yields or random supply lead times to be the "supply uncertainty". Examples of articles that adopt the first convention (either explicitly or implicitly) in a stochastic programming framework are [71], [72], [73], [74], [75], [76], [77], and [78].

Escudero et al. [71] developed a stochastic programming modeling framework for the optimization of a multi-period multi-product multi-level supply chain planning problem under demand and supply uncertainties. The supply uncertainties that they included are in the unit cost of providing raw material, supply capacity, and raw material shipment capacity. They applied their model to a problem in the automotive sector.

Alonso-Ayuso et al. [72-74] presented a two-stage stochastic 0-1 modeling approach and solution algorithms for multi-period supply chain management problems under uncertainty. The supply uncertainty is for the cost of raw materials. Santoso et al. [75] proposed a two-stage stochastic programming model and solution algorithm for solving supply chain network design problems. The supplier uncertainties in their model are the uncertainties in processing, transportation from suppliers, and supply capacity.

Li and Zabinsky [76] developed a two-stage stochastic programming model and a chance-constrained programming model that selects suppliers and determines optimal order quantities with consideration of business volume discounts. The supplier capacity was among their uncertain parameters, along with the demand, and some quality and delivery tolerances.

Bidhandi and Yusuff [77] considered a two-stage stochastic programming model for solving strategic and tactical supply chain network design problems under uncertainty. Their supply uncertainties were concerned with supply costs and capacities.

Al-E-Hashem et al. [78] used a robust multi-objective model to deal with a multi-product multi-period supply chain planning problem with multiple suppliers, multiple manufacturers, and multiple customers. Beside other sources of uncertainty, their model incorporated the cost of raw materials and transportation cost from suppliers.

The second treatment of supply uncertainty deals with random yield and/or random lead times. Random yield results in received quantities being random fractions of ordered ones [79]. The undelivered portions of ordered quantities are lost and not expected to be shipped. Some authors have recently referred to supplier random yields as stochastic supplier reliability, or supplier unreliability (see, for example, [80]). Random yield has been an active research area for multiple years. For review articles on random yield, we refer the reader to [81] and [82]. Beside the simplest form where the number of good units in a batch has a binomial distribution, Yano and Lee [81] identified two modeling approaches. In the first approach, the yield uncertainty is stochastically proportional to the quantity ordered, i.e. the quantity delivered is a random fraction of the quantity ordered. In the second one, the fraction of good units is a function of the batch size. We use the former approach in our model below. Note that random yields are not only present in the quantities ordered from suppliers, but might also in production quantities (see for example [83]). That case is out of the scope of our work here.

The lead time refers to the time elapsed from order release to order delivery [84]. Multiple authors incorporated stochastic lead times in their supply chain planning models. For example, Dolgui and Ould-Louli [84] studied a single level multi-item multi-periods supply planning problem within the materials requirement planning (MRP) framework under lead time uncertainty. Their solution approach is based on the use of an auxiliary Markov chain. Another example is the work of Abginehchi and Farahani [85]. They developed a mathematical model for multiple supplier single item inventory systems under supplier lead time uncertainty. A review of stochastic supplier lead time can be found in [86].

Gupta and Brennan [87] adopted a modeling approach that combines quantity and timing variability (i.e. random yield and lead time uncertainty) in which half of the ordered material arrives on-time, $\alpha\%$ of the other half arrives early, and the remaining

$(1-\alpha)\%$ arrives late, where $0 \leq \alpha \leq 1$. A more general approach that combines both the random yield and stochastic supply lead times can be found in Bollapragada and Rao [88]. We will explain it in detail in Section 3 below and illustrate how we generalize it.

3.3 Problem Definition and Model Description

In this Section, we start with describing the structure of the supply chain considered in our work. Next, we discuss the problem definition, model assumptions, and formulate our two-stage stochastic programming model.

The supply chain network structure considered in our model is similar to the one in the previous chapter. It is a multi-period, multi-product supply chain, consisting of suppliers, transformation facilities, and customers. Set P is the set of products, including raw material, semi-finished, or finished products. Set T is the set of time periods. Suppliers are denoted by the set S. Transformation facilities (denoted by the set TF) are facilities/factories where processing (manufacturing/assembly) and/or storage of products occur. Set K is the set of customers. Suppliers supply products (either raw materials or semi-finished products) to transformation facilities. Products are then transported from the transformation facilities to other transformation facilities if further processing or storage is needed, or to customers to fulfill their demand.

There is a bill of material (BOM) for each product. The flow of products can only occur through predefined channels from suppliers to transformation facilities, between transformation facilities, or from transformation facilities to customers. The echelon structure of the supply chain can differ by product and transformation facilities can be present at different stages of the supply chain for different products. Products can be kept in inventory at any transformation facility either before being processed or after processing. Processing, inventory, and throughput capacities are also considered in the model. All capacities can be on a product by product basis or jointly for a set of products. All transportation costs for all of the predefined transportation channels are assumed to be known. Additionally, fulfilling the demand at a later period (backordering) is allowed. The customer's demand has to be fully satisfied by the end of the planning horizon to avoid an expensive lost sales penalty. Inventory "flows" can only occur at transformation facilities and backorder "flows" can only occur at customers' sites. Backorder penalties are functions of the backorder delay which is the difference between the actual period of the demand and the fulfillment period.

As for the uncertainty, we assume that both supply and demand are random with known probability distributions. Let Ω denote the vector of the uncertain supply and demand, where $\Omega = (\Delta, D), \Delta$ corresponds to supply uncertainty, D refers to demand uncertainty, and ω is a given realization of the uncertain parameters. Before the realizations of these random parameters, the decisions of how much and when to order from each supplier need to be made. Thus, we define the first stage variable pq_{ijpt} as the purchase quantity of product $p \in P$ from supplier $i \in S$ to transformation facility $j \in TF$ in period $t \in T$, and pc_{ip} as the unit cost of product $p \in P$ ordered from supplier $i \in S$ in time period $t \in T$.

We assume that each selected supplier $i \in S$ is required to provide this purchased/ordered quantity. However, due to supplier unreliability/uncertainty, each of these selected suppliers delivers only a percentage of the ordered quantities on time, and delays the rest, according to some random reliability index. For each supplier $i \in S$ and product $p \in P$ in scenario $\omega \in \Omega$, we define the supplier uncertainty index $\Delta_{iptt'}(\omega)$ as the percentage that the supplier will deliver in period $t \in \{t', \dots, T\}$ of what he should have delivered in period $t' \in T$, where $\sum_{t \in \{t', \dots, T\}} \Delta_{iptt'}(\omega) \leq 1$. Each supplier $i \in S$ pays a penalty $ps_{iptt'}$ for each unit of product $p \in P$ delivered in period $t \in \{t' + 1, \dots, T\}$, when it should have been delivered in period $t' \in T$. We assume that suppliers never deliver orders earlier than scheduled. Notice that this can be trivially relaxed and included in the above convention. Since we are modeling the problem from the buyer's perspective, the penalty $ps_{iptt'}$ is subtracted from all the other costs in the second stage problem objective function. This way of modeling supplier uncertainty generalizes the work of Bollapragada and Rao [88] in two distinct aspects. First, in their paper, they limited their approach to receiving quantities in just three periods (one period earlier than the order period, period of the order, and one period later than the order period). In our model, we model any number of periods starting from the period of the order. Adding earlier periods is also straight forward. However, in the application that inspired this work, orders never arrive earlier than planned, so we focus only on on-time and late arrivals. Second, in our model, we generalize their approach to multi-item, multi-supplier, and multi-facility supply chains as seen above.

We also note that the lead time can include another deterministic period in addition to the stochastic one mentioned above. This former period corresponds to the time elapsed from the order placement until the scheduled order delivery. Then, our stochastic portion models the random time spent between the scheduled order delivery until the actual delivery date. We do not include the deterministic portion of the lead time in our model. In practice, users of our model can simply subtract that portion from the resulting order procurement period of our model in order to get the periods at which they should place the orders.

We define the stochastic demand of customer $k \in K$ in period $t \in T$ for product $p \in P$ in scenario $\omega \in \Omega$ as $dem_{kpt}(\omega)$, where the demand is random with known probability distributions. In the second stage, after the uncertainty is realized, the recourse actions are as follows: if suppliers supply a raw material in a quantity that is less than the needed quantity to assemble the end products that rely on that raw material, then customer backorders are created which are fulfilled from later periods or the customer demand is to be lost. However, if supply is more than needed, then it will be stored for use during later periods in either its raw material state, its semi-finished, or its finished state; depending on inventory capacities and other cost tradeoffs in the model. Outsourcing extra inventory, manufacturing, inventory, and/or transportation resources and capacities at an additional cost is another potential recourse action for that latter case. We will refer to this as outsourcing or capacity expansion interchangeably. Note that these recourse actions guarantee that our model has a complete recourse. Meanwhile, the flow of materials, inventory, and manufactured/assembled quantities across the whole supply chain are optimized through the use of second stage decision variables. Note also that other sources of uncertainty, such as uncertainty in capacities and costs, can be directly added to the model.

Our model can be used in a rolling horizon heuristic scheme in a straightforward manner (see for example [47] and [89]). Also, expanding capacities to deal with uncertainty is not uncommon in the literature. Examples of the papers focusing on this topic can be found in [90], [91], and [92]. In addition, note that the whole described problem can be directly modeled as a multi-stage programming model, and thus we are implicitly doing an approximation and simplification to two stages.

A summary of notation of all sets, parameters, and variables are given in appendix C. Given the previous problem definition, model dynamics, and notation, the resulting formulation is as follows:

min

$$\sum_{i\in S}\sum_{j\in TF}\sum_{p\in P}\sum_{t\in T} pc_{ipt}.\, pq_{ijpt} + \mathbb{E}_{\omega}[Q(pq,\omega)] \qquad 3.1$$

s.t.

$$\sum_{j\in TF} pq_{ijpt} \leq Max_{ipt} \quad \forall i\epsilon S, \forall p\in P, \forall t\epsilon T \qquad 3.2$$

$$pq_{ijpt} \geq 0 \quad \forall i\epsilon S, \forall j\in TF, \forall p\in P, \forall t\epsilon T \qquad 3.3$$

Where $Q(pq,\omega)$ is the optimal value of the following second stage problem:

Min

$$\sum_{i\in TF}\sum_{j\in D}\sum_{p\in P}\sum_{t\in T} fc_{ipt}.\, y_{ijpt}(\omega)$$

$$+\sum_{j\in TF}\sum_{p\in P}\sum_{t\in T} ic_{jpt}.\, iq_{jpt}(\omega) + \sum_{j\in TF}\sum_{p\in P}\sum_{t\in T} ac_{jpt}.\, aq_{jpt}(\omega)$$

$$+\sum_{k\in K}\sum_{p\in P}\sum_{t\in T} pn_{kpt}.\, ls_{kpt}(\omega) + \sum_{k\in K}\sum_{p\in P}\sum_{t\in T}\sum_{u\in T} bc_{kptu}.\, bq_{kptu}(\omega)$$

$$+\sum_{i\in S}\sum_{j\in TF}\sum_{p\in P}\sum_{t\in T} ccapexp_{ijpt}.\, ccExp_{ijpt}(\omega)$$

$$+\sum_{i\in TF}\sum_{j\in D}\sum_{p\in P}\sum_{t\in T} ccapexp_{ijpt}.\, ccExp_{ijpt}(\omega)$$

$$+\sum_{j\in TF}\sum_{p\in P}\sum_{t\in T} cfcapexp_{jpt}.fcapExp_{jpt}(\omega)$$

$$+\sum_{j\in TF}\sum_{p\in P}\sum_{t\in T} cicapexp_{jpt}.icapExp_{jpt}(\omega)$$

$$+\sum_{j\in TF}\sum_{p\in P}\sum_{t\in T} cacapexp_{jpt}.acapExp_{jpt}(\omega)$$

$$+\sum_{i\in S}\sum_{j\in TF}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} crc_{ijrt}.cres_{ijprt}.x_{ijpt}(\omega)$$

$$+\sum_{i\in TF}\sum_{j\in D}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} crc_{ijrt}.cres_{ijprt}.y_{ijpt}(\omega)$$

$$+\sum_{i\in TF}\sum_{j\in D}\sum_{p\in P}\sum_{t\in T} frc_{irt}.fres_{iprt}.y_{ijpt}(\omega) +$$

$$\sum_{j\in TF}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} irc_{jrt}.ires_{jprt}.iq_{jpt}(\omega) \qquad 3.4$$

$$+\sum_{j\in TF}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} arc_{jrt}.ares_{jprt}.aq_{jpt}(\omega)$$

$$+\sum_{i\in S}\sum_{j\in TF}\sum_{r\in R}\sum_{t\in T} rccapexp_{ijrt}.RccapExp_{ijrt}\,(\omega)$$

$$+\sum_{i\in TF}\sum_{j\in D}\sum_{r\in R}\sum_{t\in T} rccapexp_{ijrt}.RccapExp_{ijrt}\,(\omega)$$

$$+\sum_{j\epsilon TF}\sum_{r\in R}\sum_{t\in T} rcfcapexp_{jrt}.RfcapExp_{jrt}\,(\omega)$$

$$+\sum_{j\epsilon TF}\sum_{r\in R}\sum_{t\in T} rcicapexp_{jrt}.RicapExp_{jrt}\,(\omega)$$

$$+\sum_{j\epsilon TF}\sum_{r\in R}\sum_{t\in T} rcacapexp_{jrt}.RacapExp_{jrt}\,(\omega)$$

$$-\sum_{i\in S}\sum_{p\in P}\sum_{t'\in T}\sum_{j\in TF} ps_{iptt'}.\left(\sum_{t\in\{t'+1,\dots,|T|\}} \Delta_{iptt'}(\omega).pq_{ijpt'}\right)$$

s.t.

$$x_{ijpt}(\omega) = \sum_{t'\epsilon\{1,\cdots,t\}} \Delta_{iptt'}(\omega)\,.\,pq_{ijpt'} \quad \forall i\epsilon S, \forall j\epsilon TF, \forall p\epsilon P, \forall t\epsilon T \qquad 3.5$$

$$\sum_{j\epsilon D} y_{ijpt}(\omega) \le fcap_{ipt} + fcapExp_{ipt}(\omega) \quad \forall i\epsilon TF, \forall j \in D, \forall p\epsilon P, \forall t\epsilon T \qquad 3.6$$

$$\sum_{p\in P} fres_{iprt}\,.\left(\sum_{j\epsilon D} y_{ijpt}(\omega)\right) \le fcap_{irt} + RfcapExp_{irt}(\omega) \quad \forall i \in TF, \forall r \in R, \forall t \in T \qquad 3.7$$

$$x_{ijpt}(\omega) \le ccap_{ijpt} + ccExp_{ijpt}(\omega) \quad \forall i \in S, \forall j \in TF, \forall p \in P, \forall t \in T \qquad 3.8$$

$$\sum_{p\in P} cres_{ijprt}.x_{ijpt}(\omega) \le ccap_{ijrt} + RccapExp_{ijrt}(\omega) \quad \forall i \in S, \forall j \in TF, \forall r \in R, \forall t \in T \qquad 3.9$$

$$y_{ijpt}(\omega) \le ccap_{ijpt} + ccExp_{ijpt}(\omega) \quad \forall i \in TF, \forall j \in D, \forall p \in P, \forall t \in T \qquad 3.10$$

$$\sum_{p\in P} cres_{ijprt}.y_{ijpt}(\omega) \le ccap_{ijrt} + RccapExp_{ijrt}(\omega) \quad \forall i \in TF, \forall j \in D, \forall r \in R, \forall t \in T \qquad 3.11$$

$$iq_{jpt}(\omega) \le icap_{jpt} + icapExp_{jpt}(\omega) \quad \forall j\epsilon TF, \forall p\epsilon P, \forall t\epsilon T \qquad 3.12$$

$$\sum_{p\in P} ires_{jprt}.iq_{jpt}(\omega) \le icap_{jrt} + RicapExp_{jrt}(\omega) \quad \forall j \in TF, \forall r \in R, \forall t \in T \qquad 3.13$$

$$aq_{jpt}(\omega) \le acap_{jpt} + acapExp_{jpt}(\omega) \quad \forall j \in TF, \forall p \in P, \forall t \in T \qquad 3.14$$

$$\sum_{p\in P} ares_{jprt}.aq_{jpt}(\omega) \le acap_{jrt} + RacapExp_{jrt}(\omega) \quad \forall j \in TF, \forall r \in R, \forall t \in T \qquad 3.15$$

$$cq_{jpvt}(\omega) = 1bom_{pv}.aq_{jvt}(\omega) \quad \forall j\epsilon TF, \forall p\epsilon P, \forall v \in P, \forall t\epsilon T \tag{3.16}$$

$$\begin{aligned}\sum_{i\epsilon S} x_{ijpt}(\omega) + \sum_{i\epsilon TF} y_{ijpt}(\omega) + re_{jpt} + aq_{jpt}(\omega) + iq_{jpt-1}(\omega) - iq_{jpt}(\omega) \\ - \sum_{v\epsilon P} cq_{jpvt}(\omega) - \sum_{i\epsilon TF} y_{jipt}(\omega) = 0 \quad \forall j\epsilon TF, \forall p\epsilon P, \forall t\epsilon\{2, \dots, T\}\end{aligned} \tag{3.17}$$

$$\begin{aligned}\sum_{i\epsilon S} x_{ijpt}(\omega) + \sum_{i\epsilon TF} y_{ijpt}(\omega) + re_{jpt} + aq_{jpt}(\omega) + init\_inv_{jp} - iq_{jpt}(\omega) \\ - \sum_{v\epsilon P} cq_{jpvt}(\omega) - \sum_{i\epsilon TF} y_{jipt}(\omega) = 0 \quad \forall j\epsilon TF, \forall p\epsilon P, t = 1\end{aligned} \tag{3.18}$$

$$\begin{aligned}\sum_{j\epsilon TF} y_{jkpt}(\omega) + \sum_{u\epsilon T} bq_{kput}(\omega) + ls_{kpt}(\omega) \\ = dem_{kpt}(\omega) + \sum_{u\epsilon T} bq_{kptu}(\omega) \quad \forall k\epsilon K, \forall p\epsilon P, \forall t\epsilon T\end{aligned} \tag{3.19}$$

$$\sum_{u\in T:u<t} bq_{kptu}(\omega) \leq \sum_{j\epsilon TF} y_{jkpt}(\omega) \quad \forall k\epsilon K, \forall p\epsilon P, \forall t\epsilon T \tag{3.20}$$

$$x_{ijpt}(\omega) \geq 0 \quad \forall i\epsilon S, \forall j\epsilon TF, \forall p\epsilon P, \forall t\epsilon T \tag{3.21}$$

$$y_{ijpt}(\omega) \geq 0 \quad \forall i\epsilon TF, \forall j\epsilon D, \forall p\epsilon P, \forall t\epsilon T \tag{3.22}$$

$$ls_{kpt}(\omega) \geq 0 \quad \forall k\epsilon K, \forall p\epsilon P, \forall t\epsilon T \tag{3.23}$$

$$bq_{kptu}(\omega) \geq 0 \quad \forall k\epsilon K, \forall p\epsilon P, \forall t\epsilon T, \forall u\epsilon T \tag{3.24}$$

$$iq_{jpt}(\omega) \geq 0 \quad \forall j\epsilon TF \cup TF', \forall p\epsilon P, \forall t\epsilon T \tag{3.25}$$

$$aq_{jpt}(\omega) \geq 0 \quad \forall j\epsilon TF, \forall p\epsilon P, \forall t\epsilon T \tag{3.26}$$

$$cq_{jpvt}(\omega) \geq 0 \quad \forall j\epsilon TF, \forall p\epsilon P, \forall v \in P, \forall t\epsilon T \tag{3.27}$$

The objective function of the first stage (equation 3.1) minimizes the total cost of purchasing from all suppliers, and the expected value of the second stage problem. Constraint 3.2 puts an upper bound/capacity on the total possible purchasing quantities from suppliers, while constraint 3.3 is the non-negativity restrictions on the first stage variables.

In the second stage, after the uncertainty has been resolved, the objective function 3.4 minimizes the total costs of purchasing from all suppliers, the total costs of transportation, inventory, manufacturing/assembly, throughput, backorder penalties, lost sales penalties, capacities for all these issues (either for products or joint resource capacities among products), and capacity expansions for all these issues, again either for products or joint among them. The last term in this objective function is the penalties paid by suppliers for delayed order quantities as explained above.

Constraint 3.5 links the ordered quantity with supplier reliability to calculate the actual delivered quantities from each supplier in each period. The rest of the model contains four types of constraints: supply capacity, transformation (production or assembly) capacity, demand satisfaction, and conservation of flow at the transformation facilities. Both regular and extended capacities are modeled in each of those capacity types. Throughput capacities at transformation facilities are modeled by constraint 3.6, which models the capacity for an individual product, and constraint 3.7 that models the joint capacity. Capacities of flow/transportation from suppliers on a product by product basis are modeled in constraint 3.8 and for joint capacities are modeled in constraint 3.9. Similar capacities for the flow from transformation facilities are modeled in constraints 3.10 and 3.11, respectively. The same applies to the transformation facilities inventory capacity in constraints 3.12 and 3.13, respectively. Also, it applies to assembly/manufacturing capacities at different transformation facilities through constraints 3.14 and 3.15. Constraint 3.16 is the BOM constraint. It ensures that the correct amounts of components are consumed in the transformation facilities in order to be assembled into finished/semi-finished goods.

Constraints 3.17 and 3.18 are the conservation flow constraints that work the same way as the corresponding ones in the previous chapter (see figure 2).

Constraint 3.19 ensures that the goods delivered to a customer, lost sales, and backorders from future periods are allocated to satisfy either the demand of that period or backorders in previous periods. The conservation of flow for the customer backorders for a single product at a single customer is illustrated in figure 9. The deliveries are goods delivered by the supply chain to this customer. Backorders satisfy demand from a period later in the planning horizon to a period earlier in the planning horizon. Note that we assume that the lost sales penalty is higher than any backorder cost penalty.

Constraint 3.20 treats the so called multi-hop problem (see chapter II for a detailed explanation of this). Lastly, constraints 3.21 through 3.27 are the non-negativity constraints.

3.4 Results and Discussion

In Subsection 3.4.1, we describe the real world industrial problem that inspired our model and explain how our model can be directly applied to it. Then, we present some numerical and theoretical results in Subsection 3.4.2.

3.4.1 Wind Turbines Application

Our application is an extension to the case study in chapter II for the wind turbines subsidiary of one of the leading power generating equipment manufacturers. In our specific case, the company has 42 different projects (customers) in North America.

Our planning horizon is one year. There is a known weekly demand schedule for any of seven different models of wind turbines. Each of these models is composed of seven different components; the tower, nacelle, bearings, blades (a set of 3 blades), gearbox, generator, and rotor. These components are supplied from one of 24 different suppliers, that are located all over the globe and some of which are facilities that are owned by the company itself.

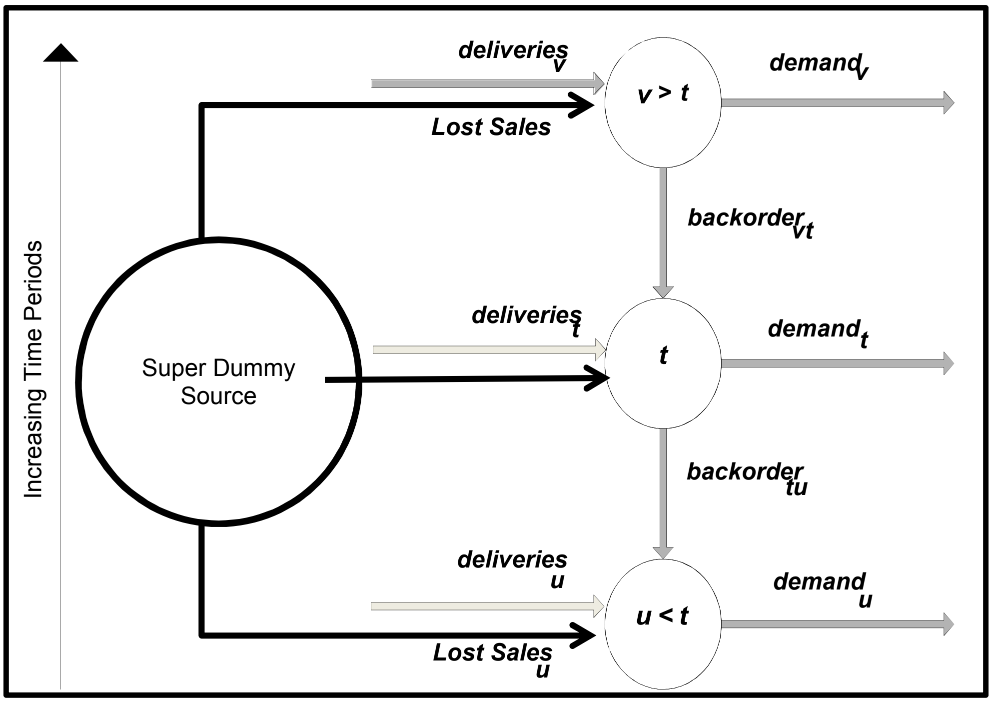

Figure 9 Conservation of Flow for Customer Backorders

Since suppliers are located over the globe; procurement decisions become vital with expensive transportation costs. In addition, supply capacities are usually tight, which was also reported by He and Chen [19]. Transportation costs from each supplier to each project site are known. Note that the strategic decisions of which suppliers to select has already been made. We consider the procurement decisions on the tactical level, i.e. how much and when to order from a known set of suppliers. Hence we do not include any supplier fixed costs in our model, nor deal with any of the methods or issues that determine supplier selection (see, for example, Ho et al. [49] for a review of these methods).

The single level BOM for each end product is known. There is a known backorder cost/penalty for the delay in meeting the demand at each specific site for each specific product. That penalty depends on the number of periods during which the product was backordered. Transportation costs along the whole supply chain are known. There is a maximum capacity for assembling each final product (wind turbine) at each project site during any specific period. There is also an upper limit on the maximum order that can be placed at any supplier for any specific product (supply capacity).

Supplier delivery is highly uncertain. Suppliers sometimes deliver parts of an order in later periods than the order period. A turbine cannot be assembled unless all its main components are present at the customer site where it will be assembled and located. Ideally, all parts for each turbine arrive on-time (i.e., at the same period of the demand) and the turbine gets assembled. However, due to supplier uncertainty, some parts might arrive late, and so arrangements for storing the parts that arrived on time need to be made. However, most of the parts, such as the blades, cannot be stored on the site. Thus, in such case, rental of an appropriate storage site near the customer sites takes place, with proper arrangement for transportation to and from that site. This corresponds directly to the storage and transportation capacity expansion in our model. In addition, when the delayed parts get delivered at later periods, assembly resource capacity might not be sufficient to assemble all turbines that are ready to be assembled in those later periods. There are two remedies/recourse actions for this case: either using additional outsourced assembly resource expansions, or storing the components of some of these turbines and assemble them at later periods. In the latter recourse action, extra outsourced inventory resources might be used as well.

In order to accommodate assembly capacity restrictions at the customer, inventory and assembly expansions, a dummy transformation facility is collocated with each customer when our model is used for this application. Flow is only allowed from each dummy transformation facility to the customer at its site with the cost of the delivery to that customer being zero.. The terms in the objective function of our model are consistent with the costs considered by the company.

The company was using spreadsheet-assisted manual planning. Management believed too many project completions, i.e. finished assemblies, were delayed and they were not satisfied with their capability to deal with supplier uncertainty. But they did not have any systematic calculation procedure to assess these beliefs. Our model can directly overcome these planning deficiencies. Next, we discuss our implementation/computational experience of our model dealing with this application, in addition to illustrating some numerical and theoretical results.

3.4.2 Numerical and Theoretical Results

The model is coded in GLPK [50], which is an open source optimization language and solved with CPLEX 12.5 [5], under Windows XP. GLPK allows reading data from different databases. We developed a database schema in Microsoft Access that contains data fields for all the model parameters and variables in different tables, and read the data from it. The machine used in the experiments has an Intel core 2 duo T7200 with 2.00 GHZ for each processor and 6 G.B. of RAM. Different instances of our case study (with 50 scenarios for the supplier uncertainty) take an average of 25 minutes for model generation including data retrieval with a standard deviation of less than a minute. The instances get solved in just 8 seconds using default CPLEX options, with a standard deviation of less than two seconds.

Our model is linear in both the first and second stages, i.e., we do not have any integrality in any of the two stages. We solve the deterministic equivalent of the stochastic model (see [13] and [14]). We do not use any decompositions or special stochastic programming solution algorithms since the solution times are small.

Next we design and report on some numerical experiments with the following aims. First, we show how instances with real data from our application can be implemented and solved successfully. Second, we empirically answer the research questions stated in the introduction of this chapter.

To study the impact of supplier reliability, we cluster the 24 suppliers into three groups: suppliers that are more reliable, suppliers that have average reliability, and suppliers that are less reliable. The most reliable suppliers deliver quantities either on-time, one period late, or two periods late. The percentage of quantities delivered in each period follows a binomial distribution with a probability that ranges linearly from 0.01 to 0.99 (with a step of 0.02) for the 50 scenarios. The same applies for moderately reliable suppliers and least reliable (most unreliable) ones, except that their quantities are delivered up to 5 and 8 periods late, respectively. Note that some orders might arrive after the last period of the considered planning horizon (52 periods). This is treated by imbedding our modeling framework in a rolling horizon heuristic. However, for the sake of comparison, we neutralize this end horizon effect by adding a number of periods with zero demand at the end of the planning horizon. We will explain this more formally in our theoretical analysis.

We use 4 different levels for the purchasing costs and two levels for the backorder costs. The levels of the purchasing costs are as follows: equal low costs for all suppliers, equal high costs for all suppliers, low costs for all suppliers but with the more reliable suppliers being more expensive, and high costs for all suppliers but with the less reliable suppliers being less expensive. The proportion of costs for the three supplier clusters in the latter two levels is 1:0.7:0.4. We use a linear delay-dependent backorder cost structure. We carried out a factorial experiment with the levels of the two factors (purchasing costs and backorder costs) as describe above. All other parameters have values that derived from the real data for our wind turbines application.

We solve three variants of the problem: the deterministic case of completely reliable suppliers, the expected/mean value problem, and the stochastic problem. By a completely reliable supplier, we mean a supplier $i \in S$ that delivers the whole amount of its orders on time, i.e. it has $\Delta_{iptt'} = 1 \ \forall p \in P, \ \forall t' \in T, \forall t \in \{t', \ldots, T\}: t = t'$ and $\Delta_{iptt'} = 0 \ \forall p \in P, \forall t' \in T, \forall t \in \{t', \ldots, T\}: t \neq t'$. Thus, the completely reliable supplier problem is the deterministic version of the problem discussed in the previous chapter of this thesis. The expected/mean value problem is the problem obtained by replacing all random variables by their expected values (see [14]).

We use the three clusters of the suppliers for the two latter variants, and the 50 scenarios for the single latter one. Tables 3 and 4 show the results of the second and third variants, respectively. In Table 3, the percentage increase of the optimal solution of the expected/mean value problem over the corresponding cases of completely reliable suppliers are presented for each experiment. In Table 4, we show the value of using the stochastic program versus the deterministic one by reporting the following percentage: (the value of the stochastic program/solution of the stochastic model) X 100%. We again refer the reader to [13] and [14] for a detailed definition of the value of the stochastic program (VSS). These percentages reach values of up to almost 20% (see column H in table 4). In addition, procurement percentages from each of the three previously described clusters are indicated for each experiment of the two variants.

Analyzing the results of the two tables, we can state the following results. First, the model always chooses the cheapest suppliers (so long as the supply capacity permits), regardless of whether these suppliers are more or less reliable, and does the rest of the planning around this choice. This can be seen from the percentages of procurement from each cluster of suppliers for the cases where purchasing costs differ from one cluster to another, either for the expected value problem or the stochastic problem. Our model allows the decision makers to select the cheapest suppliers and to adapt their other decisions to incorporate the unreliability of the selected suppliers while minimizing the overall costs.

To illustrate this with a simple example, consider the case of a supplier that is cheaper than all other suppliers and a single scenario problem where this supplier always delivers 0.3 of the ordered quantity on-time and the remaining 0.7 one period late. The optimal solution might then be one of three options. First, we could schedule an earlier delivery and keep the quantities that would arrive earlier than needed inventory. Second, we might order on-time more than the needed demand, keep in inventory the additional items that will eventually be received and use them in later periods. Third, we could order just the needed quantities on-time and do a backorder for the portion that we will receive late. This illustrates how the model could plan around the choice of the cheapest supplier(s). The stated potential solutions are illustrated below in our results and analyses as well.

Second, we can see that the optimal solution of the expected/mean value problem is always higher than or equal to that of the corresponding problem with completely reliable suppliers. This is attributed to the fact that the model has to account for the unreliability of suppliers, e.g., by using more backorders and/or inventory.

Third, in our results, whenever the backorder costs increase, the percentage increase of the solution value of the expected/mean value problem over the solution value of the corresponding problem with completely reliable suppliers gets higher because either more inventory is used, more expensive backorder is used, or a combination of both is used.

Fourth, based on our numerical results we report the following effect of supplier uncertainty on the optimal procurement decisions. When supplier uncertainty is considered (for either of the two cases), the optimal total purchased quantities might be more than what is needed to cover the demand. In our experiment, this happens only for the cases of large backorder costs and small purchasing costs. This result might be counterintuitive at first, so we formalize a theoretical analysis that explains it.

We first deal with the case of our problem with perfect information, i.e., just one scenario and start with some notation: let End_{ipt} be the last period in which a positive part of an order placed for product $p \in P$ from supplier $i \in S$ in period t will arrive. This leads to that $\Delta_{iplt}= 0 \ \forall l > End_{ipt}$. In our below analysis, we will compare total procurement in the whole planning horizon for different cases, just as we did in the numerical results. However, since *t* might be outside the current planning horizon, we add more periods with zero demand in T in order to neutralize this end horizon effect. Thus, $|T_{new}|$=$max\{|T|, \max_{i\in S,p\in P,t\in T} End_{ipt}\}$, where $|T|$ is the cardinality of the set of periods T, and $|T_{new}|$ is the cardinality of the new set of periods after adding the described periods.

We now present a simple example to show that the total ordering quantity can exceed the ordering quantity in the case of completely reliable suppliers, given that everything else in the problem remains unchanged. Suppose that there is only 1 supplier, 1 product, and a single demand for that product of m units in period 1, for a horizon of 12 periods. If the supplier is completely reliable, the optimal solution will be to order m units in period 1, if no other constraints make this an infeasible solution. Further, suppose that $\Delta_{ip11}= n, \Delta_{ip21}= 1 - n$, where $0 < n < 1$ if we are to solve the same problem with the supplier being uncertain with just one possible scenario. We do not need to add any periods here; since $\max_{i\in S,p\in P,t\in T} End_{ipt} = 2 < |T| = 12$.

One possible solution of this latter problem might be to order m units in period 1, receive nm units in period 1, use them to fulfill part of the demand, and then receive the remaining $m - nm$ units in period 2 and use those units to fulfill the remaining part of the demand of period 1 while paying a backorder penalty for delaying those units for one period. Another possible solution is to order m/n units in period 1, and therefore receive m units in this period to satisfy the whole demand on time, and receive the rest of the $\frac{m}{n}$ units (i.e., $\frac{(1-n)}{n}m$ units) in period 2. That latter amount will then be kept in inventory until the last period in the horizon. Therefore, the total ordering quantity would be greater than the case of the completely reliable suppliers. For the second solution to be feasible, a recourse action of using some capacity expansions and paying its extra costs might be required. If both solutions are feasible, the second one will be optimal if the backorder cost for backordering $m - nm$ units for 1 period is more expensive than the cost of purchasing, transporting, and storing those extra $\frac{m}{n} - m$ units, in addition to any potential capacity expansion costs.

We give sufficient conditions that prevent this behavior for the simple case of having no BOM, i.e., the purchased raw materials are the same as the end products in Appendix D.

Table 3 Results for the Expected/Mean Value Problem

A	B	C	D	E	F	G
Low	Low and Equal for All Suppliers	1.51	62.34	11.08	26.58	0.00
High	Low and Equal for All Suppliers	3.12	69.71	13.13	17.16	0.09
Low	High and Equal for All Suppliers	0.09	44.67	18.48	36.85	0.00
High	High and Equal for All Suppliers	0.27	50.99	20.03	28.98	0.00
Low	High with More Reliable Suppliers Being More Expensive	0.62	0.00	0.00	100.00	0.00
High	High with More Reliable Suppliers Being More Expensive	4.15	0.00	1.94	98.06	0.45
Low	Low with More Reliable Suppliers Being More Expensive	9.72	0.00	2.01	97.99	0.00
High	Low with More Reliable Suppliers Being More Expensive	20.79	2.10	1.83	96.06	3.71

Table 4 Results for the Stochastic Problem

A	B	D	E	F	G	H
Low	Low and Equal for All Suppliers	51.38	9.99	38.63	0.00	1.44
High	Low and Equal for All Suppliers	52.87	10.54	36.59	0.26	2.57
Low	High and Equal for All Suppliers	41.83	16.85	41.32	0.00	0.09
High	High and Equal for All Suppliers	41.38	17.17	41.44	0.00	0.19
Low	High with More Reliable Suppliers Being More Expensive	0.00	0.00	100.00	0.00	0.60
High	High with More Reliable Suppliers Being More Expensive	0.00	0.00	100.00	0.04	3.92
Low	Low with More Reliable Suppliers Being More Expensive	0.00	0.00	100.00	0.00	9.50
High	Low with More Reliable Suppliers Being More Expensive	0.00	0.03	99.97	0.69	19.55

Headings:
***A:** Backorder Cost Level. **B:** Purchasing Cost Level. **C:** Percentage Increase over Corresponding Case of Completely Reliable Suppliers. **D:** Percentage of Procurement from the Cluster of Most Reliable Suppliers. **E:** Percentage of Procurement from the Cluster of Moderately Reliable Suppliers. **F:** Percentage of Procurement from the Cluster of Least Reliable Suppliers. **G:** Percentage of Additional Procurement over Demand. **H:** VSS/Stochastic Solution.*

Chapter IV

A MODELING FRAMEWORK AND SOLUTION METHODOLOGY FOR A PRODUCTION-DISTRIBUTION PROBLEM WITH TIME-AGGREGATED QUANTITY DISCOUNTS

4.1 Introduction

One of the most important strategic decisions in supply chain planning is supplier selection [93]. It is very common in many industries that suppliers offer quantity discounts, i.e., they offer price-volume break intervals with a unit sales price if the purchased amount falls in that interval. Unit sales prices are, of course, strictly decreasing for each higher interval. If the discount is for the whole sales volume, it is called all-units discount. If it is only for the amounts in the larger interval, it is called incremental discount. See [94] and [95] for recent definitions and a review of these two quantity discounts schemes. The latter discount scheme is a much more practical situation [96], much more common in the literature, and is the one that we consider in this work.

Even though we consider an all-units quantity discounts problem, there are two major differences between the case we study and the ones being considered extensively in the literature. First, in the literature pertaining to multi-period lot sizing with all-units quantity discounts, the quantity discounts scheme is for each period. That is, the discounts are for the total order placed in that period only. Such periods are weeks or months, for a planning horizon of several days, weeks, or months. In addition, a fixed cost is paid for each chosen supplier in each period. In any later period, the buyer can choose different suppliers and pay their fixed costs for that later period. Depending on the purchased quantity in that later period, the per-unit price is again recalculated. The case studies found in the majority of the literature follow this scheme (see, for example, [94]).

However, suppliers in practice are often selected for a strategic planning period ranging from one to five years. A fixed cost is associated with selecting a supplier and a contract is signed at the beginning of that strategic planning period. The quantity discounts established in the contract are based on the aggregate annual purchasing quantities. Afterwards, individual orders are placed during each tactical planning period which can be a week or a month. The buyer pays an estimated purchasing price based on the anticipated annual purchased quantity. At the end of the year, the actual annual purchasing quantity is known, the corresponding unit purchasing price is calculated using the quantity discount scheme, and the accounts are settled through an additional payment or refund. Usually, for the latter case, the seller may credit the refund towards future purchases by the buyer. Thus, three main characteristics differentiate this research from the relevant literature:

i. We select suppliers in the beginning of the strategic planning horizon of multiple years. Selected suppliers cannot be changed in that horizon.

ii. Fixed costs are paid only once for each chosen supplier for the entire strategic planning horizon.

iii. Per-unit costs (variable costs) depend on the annual quantity purchased, i.e. the total aggregated quantity purchased throughout the whole year, while orders can be placed in every tactical time period, which is typically weeks or months.

This supplier selection and purchasing agreements exist in practical applications such as some supply chains in the food industry. In this chapter, we will consider a real-world example in this industry. To the best of our knowledge, this case has not been considered in the literature. Very few papers discuss the aggregate aspect of it such as Bassok and Anupindi [97] and Hammami et al. [98], but none considers the exact aforementioned contractual details.

Second, we consider a comprehensive multi-period, multi-product, multi-echelon supply chain planning model with production, transportation, inventory, and a recursive multi-level bill of material (BOM) considerations. To the best of our knowledge, none of the models in the literature considers such comprehensive supply chain structure with the described quantity discounts.

We formulate our problem as a mixed integer linear programming (MIP) model. Contemporary commercial MIP solvers require many hours of computation time to find even a feasible solution for realistic problem instances. We develop an algorithm that constructs an initial solution and three iterative algorithms that improve that initial solution. We compare the performance of these algorithms and show their computational efficiency.

The contributions of this chapter are therefore twofold. First, we define and formulate a comprehensive supply chain planning model with a novel realistic time-aggregated quantity discounts scheme for suppliers. Second, we develop customized solution algorithms to solve our highly intractable model and show the computational efficiency of using these algorithms within a food supply chain real-world application.

The rest of this chapter is organized as follows: In Section 4.2, we review the related literature. The detailed problem definition and model development is presented in Section 4.3. In Section 4.4, the solution methodology is explained, while the design of experiments and numerical results are shown in Section 4.5.

4.2 Literature Review

In our problem, we assume that a set of potential suppliers has already been identified. Thus, the decision of selecting which suppliers to sign contracts with/purchase products from is now based solely on costs (procurement, transportation costs from the suppliers, and other related costs in other parts of the supply chain). Therefore, our intention is not to review general supplier selection methods and criteria. For that, we refer the reader to the review articles [49, 93, 96, 99, 100].

We focus on the literature of supplier selection with quantity discounts. Although researchers have been studying this problem for a few decades, most papers in the literature focus on supplier selection within the lot sizing (production planning) problem rather than supply chain (production-distribution) planning [96, 98].

In their review, Benton and Park [101] classify lot sizing problems under quantity discounts into non time-phased demand (single period models) and time-phased demand (multi-period models). They further classified each scheme into all-units discounts and incremental discounts. Finally, they classified the quantity discount planning problems based on the buyer-only or the buyer-supplier perspectives. Our work falls in the time-phased demand, all-units, buyer's perspective branch.

For multi-item and multi-period models, two other variants are considered in the literature: business volume discounts and bundle discounts [96]. In business volume discounts, multiple products can be purchased from each supplier and the discount is based on the total dollar amount of each specific order rather than the quantity of the individual products or the number of products purchased. Some researchers refer to this problem as the total quantity discount (see, e.g., [102]). Examples of recent papers dealing with it are those of Goossens et al. [103], Manerba and Mansini [102], Mansini et al. [104], and Manerba and Mansini [105]. Goossens et al. [103] studied multiple variants of this problem. They used three exact algorithms based on branch-and-bound and branch-and-cut techniques to solve their problems and their variants. They reported results for instances involving 50 suppliers and 100 products. Note that in this variant, the discount is aggregated across the products. For our problem, the aggregation for the discount is across the annual periods.

The bundle discounts is a scheme in which the price of a product depends on the quantities ordered from a set of related products so that everything is sold as a bundle [96]. Examples of works modeling this scheme can be found in [106-108].

Very few papers consider a scheme close to the one we consider in this chapter. Bassok and Anupindi [97] formulated a stochastic dynamic programming model for the analysis of supply contracts under demand uncertainty. In these contracts, suppliers offer quantity discounts for the aggregate order quantity over the planning horizon. The authors called these quantities the "committed" quantities. Their problem structure and numerical examples had a single product and a single supplier.

Hammami et al. [98] formulated a two-stage stochastic programming model for a supplier selection global supply chain problem with uncertain fluctuations of currency exchange rates and time-aggregated quantity discounts. They mentioned that this type of quantity discounts is common in the automotive industry. In the case study they presented, they used 4 periods (4 quarters of a one year planning horizon). Moreover, they considered 4 suppliers with 4 quantity discount intervals each. For this small instance, they were able to solve the deterministic equivalent of their problem to optimality in reasonable time using a commercial solver. They indicated that when the instance size increases, their model becomes more difficult to solve. They also reported that for larger instances, one would need to develop customized solution algorithms. Our research develops a family of such customized algorithms. The size of the instances we solve using our customized algorithms are more than 10 times larger than the largest instances they have solved. We solve instances with an average of 350 suppliers each having 6 quantity discount intervals.

To solve our large-scale MIP model, we develop customized solution algorithms that use a MIP-based local search technique. In this local search (LS), successive reduced MIP problems are solved iteratively. Some authors refer to it as integer programming (ILP)-based LS (see, e.g., [109, 110]). The reduced problems are typically formed by fixing some variables at their values in the previously found feasible solution and letting the solver determine the remaining variables. This methodology has been successfully used in recent years, especially for logistics problems. Hewitt et al. [111] solved the fixed charge network flow problem using what they called a mix of mathematical programming techniques and heuristic search techniques. To construct good feasible solutions, they used an integer programming LS algorithm on the arc-based formulation of the problem. They showed the efficacy of the proposed method in their numerical results.

Erera et al. [112] also used an integer programming-based iterative approach that searches a large neighborhood for the service network design problem faced by less-than-truckload freight transportation carriers. They used real-world data from a large U.S. carrier and showed how their method can generate significant cost savings. Along with an approximate dynamic programming approach, Papageorgiou et al. [113] used a MIP-based LS for a deterministic long-horizon maritime inventory routing problem. They showed that the latter approach significantly outperformed a leading commercial solver. Other papers that use MIP-based LS for logistics planning problems can be found in [109, 110, 114-117].

In the context of quantity discounts, Manerba and Mansini [102] studied a capacitated total quantity discounts problem and developed an exact algorithm to solve it. Although they do not refer to it as such, embedded in the algorithm is a MIP-based LS that uses a neighborhood $N(s, h)$. This neighborhood consists of all solutions that differ from solution s by at most h selected intervals. In [105], they enhanced that LS algorithm and introduced an ILP refinement procedure. They used the same idea of the neighborhoods of their previous work. Mansini et al. [104] extended the same problem to include truckload shipping, and used a LS algorithm that depends on rounding fractional variables after solving the linear programming relaxation.

Two main points distinguish our problem from that body of research. First, our quantity discounts scheme is very different from theirs. Second, the neighborhoods we adopt for our LS algorithms are different as well; since our neighborhoods are derived from turning suppliers and/or raw materials on and off. This will be illustrated in detail in Section 4.4 below. We discuss the detailed problem definition and develop our model in the next Section.

4.3 Problem Definition and Model Description

The supply chain considered in this work is similar in structure to the one in chapter II, except that backorders are not allowed. It consists of suppliers, transformation facilities, and customers. Each supplier, if selected, supplies one of the raw materials that are shipped to the transformation facilities for processing. A contract is signed for each selected supplier and a fixed cost is paid just once in the beginning of the planning horizon to cover this strategic horizon (e.g., 5 years). Then, there is a unit cost for each purchased unit that follows the scheme discussed above for each year among the years of the contract. There are also bounds on the minimum and maximum monthly order quantities from each chosen supplier.

Purchased raw materials are processed in processing facilities to produce intermediate and end products that are ordered by customers, according to a multi-product general form BOM. That demand is either fulfilled on time or is partially or completely lost. There is a penalty for each unfulfilled unit (penalty for lost sales). We assume that this lost sales cost is higher than any other cost in the whole supply chain. Products can be stored in inventory at any of the facilities, either in raw material, semi-finished, or end-product form. That might be needed due to manufacturing, inventory, throughput, and/or inventory resource capacity restrictions preventing the manufacturing and delivery of end products at the same period of the demand. All the aforementioned restrictions are included in our model. Each of these capacity restrictions is either for each product separately or joint among products. There is a cost associated with the utilization of each of these resources, a cost for transportation along the whole supply chain, a cost for holding inventory, another for manufacturing any semi-finished or end-product, and one associated with the throughput of products out of the transformation facilities. We minimize all these costs in the objective function of our mode below along with the fixed supplier selection cost, purchasing costs, and the total penalty of unfulfilled demand.

Given this structure, capacities, and dynamics, our problem can be defined as selecting the supplier(s) to buy each raw material from for the whole strategic planning horizon, and then determining the production plan. That plan consists of the quantities to be ordered from each supplier (which will determine which quantity discount interval is chosen each year), quantities assembled/manufactured quantities, inventory quantities for each product at each transformation facilities, and flow quantities between each two nodes in the supply chain, in addition to lost sales.

We formulate the problem as a MIP model. We assume that demand and capacities are known in detail (per month) only for the first year. This is not uncommon in practical situations where detailed demand and other parameters are not known for later years. Thus, we model the 12 months of the first year of the contract (high fidelity) while the remaining years will each be modeled as an aggregate period (low fidelity). In practice, the decisions of the first year will be implemented, after which the model can be solved again before the second year begins, fixing the selected suppliers in the first year, and so on. This makes it similar to a rolling horizon heuristic scheme for related problems (see for example [47, 89]) and justifies this form of modeling. It also treats the issue of ending inventory, especially in earlier years (year 1) as information of later years is included. That is because the ending inventory of the first year is optimized and held to the second year, and so on. The ending inventory in the last year does not matter because it is far in the future and the solution will not be implemented now with this initial rolling horizon result. Also, the modeling of the last year was in the coarse tuning part explained above since no detailed information for later years is available at the time being anyway. We refer the reader to [118] for a more detailed explanation of the latter ending inventory part. We next define the notation for our model then present the mathematical formulation.

4.3.1 Sets

S Set of suppliers.

RM Set of raw materials

IM Set of intermediate products

FP Set of final products

P Set of products. $P = RM \cup IM \cup FP$

TF Set of transformation facilities.

T Set of all time periods. $T = \{1,2,\cdots,12,13,\cdots,|T'|\}$.

T' Set of annual time periods. $T' = \{1,13,14,\cdots,|T'|\}$.

$\bar{T}$ Set of monthly time periods in the first year. $\bar{T} = \{1,\cdots,12\}$.

K Set of customers.

B_{ip} Set of intervals for supplier $i \in S$ supplying product $p \in P$.

D Set of destinations, $D = TF \cup K$.

R Set of resources.

4.3.2 Parameters

sc_{ip}	Fixed cost paid if supplier $i \in S$ is chosen to provide product $p \in RM$.
pc_{ipb}	Purchasing cost of product $p \in RM$ from supplier $i \in S$ on interval $b \in B_{ip}$.
tc_{ijpt}	Transportation cost from origin $i \in S \cup TF$ to destination $j \in D$ for each unit of product $p \in P$ in period $t \in T$.
$fc_{jpt}, ac_{jpt}, ic_{jpt}$	Throughput, assembly/manufacturing, and inventory costs, respectively, per unit of product $p \in P$ at transformation facility $j \in TF$ in period $t \in T$.
$frc_{jrt}, arc_{jrt}, irc_{jrt}$	Unit resource cost of resource $r \in R$ for flow, assembly/manufacturing, and inventory, respectively, at transformation facility $j \in TF$ in period $t \in T$.
$fres_{jprt}, ares_{jprt}, ires_{jprt}$	Units of resource $r \in R$ consumed by one unit of product $p \in P$ shipped, assembled/manufactured, and stored, respectively, at transformation facility $j \in TF$ in period $t \in T$.
$Penalty$	Penalty for each unfulfilled unit of customer demand.
LB_{ipb}	Minimum annual purchasing quantity of product $p \in RM$ from

	supplier $i \in S$ to get the unit price of interval $b \in B_{ip}$.
UB_{ipb}	Maximum annual purchasing quantity of product $p \in RM$ from supplier $i \in S$ to get the unit price of interval $b \in B_{ip}$, where $UB_{ipb} > LB_{ipb}\ \ \forall b \in B_{ip}$, and $UB_{ipb} = LB_{ipb+1}\ \ \forall b \in B_{ip} \setminus \{\lvert B_{ip} \rvert\}$. $\lvert B_{ip} \rvert$ is the cardinality of set B_{ip}.
Min_{ipt}	Minimum monthly purchasing quantity of product $p \in RM$ from supplier $i \in S$ in period $t \in \bar{T}$.
Max_{ipt}	Maximum monthly purchasing quantity of product $p \in RM$ from supplier $i \in S$ in period $t \in \bar{T}$. Typically, $\frac{UB_{ip\lvert B_{ip} \rvert}}{12} \le Max_{ipt} \ll UB_{ip\lvert B_{ip} \rvert}$.
$init\_inv_{jp}$	Initial inventory of product $p \in P$ at transformation facility $j \in TF$.
$1bom_{pv}$	Amount of product $p \in RM \cup FP$ needed to manufacture/assemble one unit of product $v \in FP$
dem_{kpt}	Demand of product $p \in FP$ for customer $k \in K$ in period $t \in T$.
$icap_{jpt}$	Inventory capacity for product $p \in P$ at transformation facility $j \in TF$ in period $t \in T$.
$icap_{jrt}$	Capacity of inventory resource $r \in R$ at transformation facility $j \in TF$ in period $t \in T$.
$fcap_{jpt}, acap_{jpt}$	Throughput, and assembly/manufacturing capacities, respectively, for product $p \in P$ at transformation facility $j \in TF$ in period $t \in T$.

$fcap_{jrt}$ Capacity of throughput resource $r \in R$ at transformation facility $j \in TF$ in period $t \in T$.

$acap_{jrt}$ Capacity of assembly/manufacturing resource $r \in R$ at transformation facility $j \in TF$ in period $t \in T$.

4.3.3 Decision Variables

s_{ip} $\begin{cases} 1 \text{ if supplier } i \in S \text{ is chosen to supply product } p \in RM. \\ 0 \text{ otherwise.} \end{cases}$

z_{ibpt} $\begin{cases} 1 \text{ if interval } b \in B_{ip} \text{ for supplier } i \in S \text{ and product } RM \in P \text{ is chosen in period } t \in T'. \\ 0 \text{ otherwise.} \end{cases}$

pq_{ijptb} Quantity of raw material $p \in RM$ purchased from supplier $i \in S$ on interval $b \in B_{ip}$ and transported to facility $j \in TF$ in period $t \in T$.

y_{ijpt} Quantity of product $p \in P$ transported from transformation facility $i \in TF$ to another transformation facility or customer $j \in D\backslash\{i\}$ in period $t \in T$. Note that $p \in IM \cup FP$ if $j \in TF$, while $p \in FP$ if $j \in K$.

ls_{kpt} Lost sales quantity out of demand of product $p \in FP$ at customer $k \in K$ in period $t \in T$.

iq_{jpt} Inventory quantity of product $p \in P$ held in transformation facility $j \in TF$ at the end of period $t \in T$.

aq_{jpt} Assembled/Manufactured quantity of product $p \in IM \cup FP$ at transformation facility $j \in TF$ in period $t \in T$.

cq_{jpvt} Quantity of product $p \in P$ used to assemble/manufacture product $v \in RM \cup IM$ at transformation facility $j \in TF$ in period $t \in T$.

4.3.4 Model Formulation

Given the previous problem definition, model dynamics, and notation, the resulting formulation is as follows:

min

$$\sum_{i\in S}\sum_{p\in RM}\sum_{b\in B_{ip}} sc_{ip}.s_{ip} + \sum_{i\in S}\sum_{j\in TF}\sum_{p\in RM}\sum_{t\in T}\sum_{b\in B_{ip}} pc_{ipb}.pq_{ijptb}$$
$$+\sum_{i\in S}\sum_{j\in TF}\sum_{p\in RM}\sum_{t\in T}\sum_{b\in B_{ip}} tc_{ijpt}.pq_{ijptb} + \sum_{i\in TF}\sum_{j\in D\setminus\{i\}}\sum_{p\in IM\cup FP}\sum_{t\in T} tc_{ijpt}.y_{ijpt}$$
$$+\sum_{j\in TF}\sum_{p\in IM\cup FP}\sum_{t\in T} ac_{jpt}.aq_{jpt} + \sum_{j\in TF}\sum_{p\in IM\cup FP}\sum_{r\in R}\sum_{t\in T} arc_{jrt}.ares_{jprt}.aq_{jpt}$$
$$+\sum_{j\in TF}\sum_{p\in P}\sum_{t\in T} ic_{jpt}.iq_{jpt} + \sum_{j\in TF}\sum_{p\in P}\sum_{r\in R}\sum_{t\in T} irc_{jrt}.ires_{jprt}.iq_{jpt} \qquad 4.1$$
$$+\sum_{i\in TF}\sum_{j\in D\setminus\{i\}}\sum_{p\in P}\sum_{t\in T} fc_{jpt}.y_{ijpt} + \sum_{i\in TF}\sum_{j\in D\setminus\{i\}}\sum_{p\in P}\sum_{t\in T} frc_{irt}.fres_{iprt}.y_{ijpt}$$
$$+\sum_{k\in K}\sum_{p\in FP}\sum_{t\in T} Penalty.ls_{kpt}$$

s.t.

$$\sum_{b\in B_{ip}} z_{ibpt} = s_{ip} \qquad \forall i \in S, \forall p \in RM, \forall t \in T' \qquad 4.2$$

$$LB_{ipb}.z_{ipb1} \leq \sum_{j\in TF}\sum_{t\in \bar{T}} pq_{ijptb} \leq UB_{ipb}.z_{ipb1} \qquad \forall i \in S, \forall p \in RM, \forall b \in B_{ip} \qquad 4.3$$

$$Min_{ipt}.s_{ip} \leq \sum_{j\in TF}\sum_{b\in B_{ip}} pq_{ijptb} \leq Max_{ipt}.s_{ip} \qquad \forall i \in S, \forall p \in RM, \forall t \in \bar{T} \qquad 4.4$$

$$LB_{ipb}.z_{ipbt} \leq \sum_{j \in TF} pq_{ijptb} \leq UB_{ipb}.z_{ipbt} \qquad \forall i \in S, \forall p \in RM, \forall b \in B_{ip}, \forall t \in T'\backslash\{1\} \qquad 4.5$$

$$\sum_{i \in S} \sum_{b \in B_{ip}} pq_{ijptb} + \sum_{i \in TF} y_{ijpt} + aq_{jpt} + init\_inv_{jp} - iq_{jpt} - \sum_{v \epsilon P} cq_{jpvt} - \sum_{i \epsilon D \backslash \{j\}} y_{jipt} = 0 \qquad \forall j \epsilon TF, \forall p \epsilon P, t = 1 \qquad 4.6$$

$$\sum_{i \in S} \sum_{b \in B_{ip}} pq_{ijptb} + \sum_{i \in TF} y_{ijpt} + aq_{jpt} + iq_{jpt-1} - iq_{jpt} - \sum_{v \epsilon P} cq_{jpvt} - \sum_{i \epsilon D \backslash \{j\}} y_{jipt} = 0 \qquad \forall j \epsilon TF, \forall p \epsilon P, \forall t \epsilon T \backslash \{1\} \qquad 4.7$$

$$cq_{jpvt} = 1bom_{pv}.aq_{jvt} \qquad \forall j \epsilon TF, \forall p \epsilon RM \cup IM, \forall v \in (IM \cup FP) \backslash \{p\}, \forall t \epsilon T \qquad 4.8$$

$$\sum_{i \epsilon TF} y_{ikpt} + ls_{kpt} = dem_{kpt} \qquad \forall k \epsilon K, \forall p \epsilon FP, \forall t \epsilon T \qquad 4.9$$

$$iq_{jpt} \leq icap_{jpt} \qquad \forall j \in TF, \forall p \in P, \forall t \in T \qquad 4.10$$

$$\sum_{p \in P} ires_{jprt}.iq_{jpt} \leq icap_{jrt} \qquad \forall j \in TF, \forall r \in R, \forall t \in T \qquad 4.11$$

$$aq_{jpt} \leq acap_{jpt} \qquad \forall j \in TF, \forall p \in IM \cup FP, \forall t \in \bar{T} \qquad 4.12$$

$$\sum_{p \in IM \cup FP} ares_{jprt}.aq_{jpt} \leq acap_{jrt} \qquad \forall j \in TF, \forall r \in R, \forall t \in \bar{T} \qquad 4.13$$

$$\sum_{j \in D} y_{ijpt} \leq fcap_{jpt} \qquad \forall i \in TF, \forall p \in P, \forall t \in \bar{T} \qquad 4.14$$

$$\sum_{p\in P}\sum_{i\in D} fres_{jprt}.\, y_{jipt} \le fcap_{jrt} \qquad \forall j \in TF\backslash\{i\}, \forall r \in R, \forall t \in \bar{T} \qquad 4.15$$

$$s_{ip} \in \{0,1\} \qquad \forall i \in S, \forall p \in RM \qquad 4.16$$

$$z_{ibpt} \in \{0,1\} \qquad \forall i \epsilon S, \forall p \epsilon RM, \forall b \in B_{ip}, \forall t \in T' \qquad 4.17$$

$$pq_{ijptb} \ge 0 \qquad \forall i \in S, \forall j \in TF, \forall p \in RM, \forall t \in T, \forall b \in B_{ip} \qquad 4.18$$

$$y_{ijpt} \ge 0 \qquad \forall i \epsilon TF, \forall j \epsilon D\backslash\{i\}, \forall p \epsilon P, \forall t \epsilon T \qquad 4.19$$

$$ls_{kpt} \ge 0 \qquad \forall k \epsilon K, \forall p \epsilon FP, \forall t \epsilon T \qquad 4.20$$

$$iq_{jpt} \ge 0 \qquad \forall j \epsilon TF, \forall p \epsilon P, \forall t \epsilon T \qquad 4.21$$

$$aq_{jpt} \ge 0 \qquad \forall j \epsilon TF, \forall p \epsilon IM \cup FP, \forall t \epsilon T \qquad 4.22$$

$$cq_{jpvt} \ge 0 \qquad \forall j \epsilon TF, , \forall p \epsilon RM \cup FP, \forall v \in P, \forall t \epsilon T \qquad 4.23$$

Objective function 4.1 minimizes the total supply chain costs described before. Constraint 4.2 ensures that if a supplier is chosen to provide a certain product, only one interval would be chosen each year. Constraint 4.3 defines the bounds on the annual purchased quantities shipped to all facilities, for each interval, for the first year. Constraint 4.5 does the same, but for all other years. Constraint 4.4 defines the monthly lower and upper bounds on purchasing quantities from selected suppliers in the first year. We remind the reader that we model the monthly details only in the first year.

Constraints 4.6 and 4.7 model the same transformation-space-time conservation of flow that we explained in the previous two chapters.

Constraint 4.8 is the BOM constraint. It necessitates that the correct amounts of components are consumed in the transformation facilities in order to be assembled into finished/semi-finished goods. Constraint 4.9 ensures that customer demand is either fulfilled on time or it is considered as lost sales.

There are three types of capacity restrictions at transformation facilities: inventory, assembly/manufacturing, and throughput capacities. For each of these, the restriction can be for each separate product, or it could be a joint one among different products competing for the available resources. Constraints 4.10, 4.12, and 4.14 represent the former case for inventory, assembly/manufacturing, and throughput, respectively, while constraints 4.11, 4.13, and 4.15 represent the latter case for the three capacities, respectively as well. Inventory capacity restrictions apply for the ending inventory in the first year and onwards. However, assembly/manufacturing and throughput restrictions are per month, and are therefore modeled for the first year only.

Constraints 4.16 and 4.17 put the integrality restrictions on the binary variables, while constraints (4.18-4.23) are the non-negativity constraints for the rest of the variables. Note that the supplier-product selection binary variables s_{ip} are auxiliary variables, that would be substituted in the pre-solve of any commercial MIP solver. However, these variables will be helpful in our solution methodology described in the next Section.

4.4 Solution Methodology

We first note that different users that might not have any formal knowledge in operations research usually want to quickly experiment solving multiple instances of planning models like ours with different scenario/parameter testing [113]. Hence, the focus of our algorithms below is to generate high quality solutions quickly rather than focusing on proving optimality. This could also be beneficial when extending our problem to include uncertainty; as in this case our model could be a sub-problem within a decomposition framework of a stochastic model with recourse (See [13, 14] for an overview of stochastic programming with recourse).

Our model is very challenging to solve. Leading MIP commercial solvers, such as CPLEX [5] and Gurobi [6], take an average of 40 minutes to get feasible solutions that are within a 90% relative gap from the best known objective function values for realistically-sized instances like the ones discussed in Section 4.5 below. They also take more than 60 minutes on average to get solutions that are within a 40% for such gap. Note that we are relating the gaps to the best known feasible solutions since our main objective is to find good feasible solutions quickly (see [113] for another similar example to this). That is also why we focus here on primal heuristics and a meta-heuristic that achieve this goal. This relative gap is defined as: $Relative\ Gap = \left(\frac{Z_{method} - Z_{best}}{Z_{method}}\right) * 100\%$, where Z_{best} is the objective function value of the best known feasible solution, while Z_{method} is the best objective function value obtained by a certain method that can either be a commercial solver or one of our algorithms that we discuss below.

We first present an algorithm that constructs an initial feasible solution for our model. Then, we develop two different MIP-based LS algorithms that iteratively improve this initial solution. We show and justify the idea behind choosing the neighborhood to explore in each algorithm. Lastly, we combine the two neighborhoods in a VND framework (see [16, 17] for a detailed illustration of VND).

4.4.1 Initial Solution Construction

For this initial solution, we first aggregate the quantity discount intervals for each supplier into a single interval. The lower and upper bounds of this interval are the lower bound of the original lowest interval and the upper bound of the original highest interval of the supplier, respectively. The unit cost on that single interval is the average unit cost over all intervals of the supplier. Consequently, the only integer variables remaining in this aggregated model are the supplier variables (s_{ip}). However, the problem might still be hard to solve to provable optimality. Given that we want our initial solution to be constructed quickly, we relax these variables and solve the linear programming relaxation of this described problem. Then, we use this solution to construct a feasible solution to the original MIP as follows: Any s_{ip} variable that has a value greater than zero in this solution gets its corresponding supplier set to open. The chosen interval each year for these suppliers is the one that contains the corresponding purchased annual quantity. The resulting feasible solution can be used as a warm start for the commercial solvers or for the algorithms we will develop next.

4.4.2 Supplier Selection MIP-Based Local Search Algorithm

The underlying idea behind this algorithm is the following: If we turn a potential supplier off, i.e., set its corresponding binary variable to zero, then all of the integer variables corresponding to its intervals in all years will be forced to zero as well. If we do so for many of the potential suppliers, the feasibility space can then be significantly reduced. Furthermore, we can still turn some of their intervals to zero and let the solver choose among the remaining intervals for the suppliers that we will not turn off. This reduces the search space even more. The solver can get good feasible solutions, or even the optimal solution, for the resulting reduced sub-problem quickly. Note that these solutions are feasible for the overall problem by construction.

We also observed that, for each raw material, only a few of the potential suppliers are set to one in the good solutions. Therefore, if we begin with closing all suppliers except just one and open other suppliers when needed in later iterations of the LS, we can get to these good solutions quickly. By "closing" a supplier, we mean that its corresponding binary variable s_{ip} is set to zero. By "making a supplier available" below, we mean setting the lower and upper bounds of its corresponding binary variable to zero and one, respectively, and letting the solver choose whether to open it or not. Whenever we first "make a supplier available", we only make its first three intervals available. Then, in next iterations, we close the interval lower than the chosen interval in the previous solution and make the two higher ones available. The solver would choose higher intervals of the chosen supplier(s) if the demand is too high to be covered only by that supplier(s). So, we make a new supplier available for any raw material when the previously chosen supplier gets one of its two highest intervals chosen for any of the years. Note that we implicitly assume that each supplier provides at least 4 intervals, which is the case for the realistic instances we describe in Section 4.5.

We pre-define the sequence for making the suppliers available. Multiple sorting criteria can be used. For instance, we can sort suppliers simply by the descending order of their fixed costs, by the average unit cost among all intervals, or just in a random order.

In the numerical results Section below, we present a more detailed sorting criterion that we found to be empirically useful. The pseudo code of this LS is detailed in Algorithm 2. The stopping rule that we adopt here, and in the following algorithms, is either some time limit or not getting an improvement for n successive iterations of the algorithm.

Algorithm 2 Supplier Selection MIP-Based Local Search Algorithm

Initialize:
Choose a sorting criteria for suppliers of each raw material.
for each raw material **do**
  Define set $S_p = \{i : i \in S \ and \ supplier \ i \ provides \ raw \ material \ p \in RM\}$
  Sort the elements of each set S_p according to described criteria.
  Set $s_{ip} \ to \ 1 \ \forall p \in RM : i \ is \ the \ first \ element \ in \ S_p$.
  Set all intervals of supplier i to zero, except the first three intervals. Do this for each of the 5 years.
  Define $Current_p \leftarrow i$.
  $S_p \leftarrow S_p \backslash \{i\}$.

end for
Solve the MIP with a time limit of n seconds or m feasible solutions.
if a solution is found **then**
  $BestSolutionValueSupplierLS \leftarrow the \ objective \ function \ of \ that \ solution$.
  $BestSolutionSupplierLS \leftarrow the \ variable \ values \ of \ that \ solution$.
end if

while Stopping condition not met **do**
  Fix all variables to their values in the current solution.
  for each raw material **do**
    if $Current_p$ has any of its 2 highest intervals set to 1 for any of the 5 years **then**
      if $S_p \neq \phi$ **then**
        Set the bounds on supplier i , $where \ i \in S_p$:
          $i \ is \ right \ after \ Current_p \ in \ S_p \ to \ be \ between \ 0 \ and \ 1$.
        Set all intervals of supplier i to zero, except the first three intervals. Do this for each of the 5 years.
        $Current_p \leftarrow i$.
        $S_p \leftarrow S_p \backslash \{i\}$.
      end if
    else
      Suppose chosen interval is k^{th} interval, set $k^{th} - 1$ interval to zero if $k > 1$, and each of $k^{th} + 1 \ and \ k^{th} + 2$ intervals to be between 0 and 1. Do this for each of the 5 years.
    end if
  end for
  Solve the MIP with a time limit of n seconds or m solutions.
  if a solution is found **then**
    $BestSolutionValueSupplierLS \leftarrow the \ objective \ function \ of \ that \ solution$.
    $BestSolutionSupplierLS \leftarrow the \ variable \ values \ of \ that \ solution$.
  end if
end while
Return: $BestSolutionValueSupplierLS$ and $BestSolutionSupplierLS$.

Another interesting observation we found is that even if multiple suppliers can be individually used to fulfill the need of a specific raw material, the optimal solution (or the good feasible solutions) might involve selecting more than one of these suppliers. That is because it might be more economical to use some intervals of one supplier in some years and other intervals of another supplier in the other years. Doing so might still be more economical even after putting in consideration that we will have to pay the fixed cost for all these suppliers. That is another reason why the underlying idea behind this LS works well in practice.

Note that the way we construct this algorithm (and the two coming ones) ensures that the feasible solution obtained from any iteration is feasible for the next iteration. Thus, we do not need any additional steps to verify or restore feasibility nor would we ever run into infeasibility issues. That is also why we never get worsening solutions.

4.4.3 Raw Material Separation MIP-Based Local Search Algorithm

In the previous neighborhood, we make at most two suppliers available simultaneously for each raw material. Because if we make most of the suppliers available, then the reduced search space would still be too big to explore quickly. However, because only a limited number of the potential suppliers are made available each time, the above algorithm might get trapped in local optima quickly. Therefore, we next introduce another neighborhood that explores all potential suppliers simultaneously but for a subset of the raw materials. Note that we cannot explore too many raw materials at once in order to still have a reduced searchable solution space. Consequently, this can be considered as a complementary neighborhood to the previous one, where in the first one we explore a subset of the suppliers for all raw materials while in this one we explore all suppliers but for a subset of the raw materials.

Hence, at each iteration, we only solve for all suppliers of the subset of raw materials of this iteration and fix all suppliers and intervals for all other raw materials. The questions that arise here are: How do we choose the size of the clusters? And how do we cluster the raw materials in each non-intersecting subset?

Since the larger the cluster size, the more the chance that one might get improving solutions, clusters better be as large as possible. However, if the clusters are too large, the sub-problem in each iteration might still be too hard to solve. Thus, we use the largest possible equal cluster size that makes the sub-problems still solvable to optimality or sub-optimality within the chosen iteration time limit.

As for how to cluster the raw materials, we suggest three ideas. First, we can cluster the raw materials that are required to produce each semi-finished or finished products together as much as possible, and then put the remaining ones at random in the remaining clusters. Second, we could sort the raw materials according to decreasing demand of the products that these raw materials are used in manufacturing. We then fill the clusters with the raw materials in that sorted order. In the first pass of our algorithm, we can explore the clusters in that sequence instead of choosing which ones to pursue at random in each iteration. In later passes, we can then return to the random choice of which clusters to solve for. The idea behind this sorting is that a higher demand corresponds to higher purchased quantities and thus higher total purchasing costs as a percentage of the overall objective function. Therefore, one can hope for higher improvements early on if this idea is adopted. Third, we can just cluster the raw materials at random into the equal clusters. The pseudo code of this second LS is given in Algorithm 3.

Algorithm 3 Raw Material Separation MIP-Based Local Search Algorithm

Initialize:
Choose a clustering criteria for raw materials in set RM.
Construct the initial solution using the idea in subsection 4.4.1.
Let $BestSolutionValueMaterialLS$ be the objective function value of this initial solution.
Let $BestSolutionMaterialLS$ be the variable values of this initial solution.
Divide the raw materials in set RM into N non-intersecting clusters , each with M raw materials, except the last one in case that $|RM|\%M > 0$, where $|RM|$ is the cardinality of the set RM. Thus, $N = \left\lceil \frac{RM}{M} \right\rceil$.
Define set $ALL = \bigcup_{i=1}^{N} C_i$.

while Stopping condition not met **do**
  Select cluster C_i at random from set ALL.
  Fix all supplier and interval binary variables not in that cluster to their values in the current solution.
  if $ALL \backslash C_i = \phi$ **then**
    $ALL = \bigcup_{i=1}^{N} C_i$.
  else
    $ALL \leftarrow ALL \backslash C_i$.
  end if
  Solve the MIP with a time limit of n seconds or m solutions.
  if a new improved solution is found **then**
    $BestSolutionValueMaterialLS \leftarrow the\ objective\ function\ value\ of\ that\ solution.$
    $BestSolutionMaterialLS \leftarrow the\ variable\ values\ of\ that\ solution.$
  endif
end while
Return: $BestSolutionValueMaterialLS$ and $BestSolutionMaterialLS$.

4.4.4 Variable Neighborhood Descent Algorithm

The previous two algorithms are heuristics that can get trapped in local optima. In an attempt to escape local optima, and since the two neighborhoods we described are different and complementary, we combine them into a VND meta-heuristic scheme.

We use the same initial solution as our first algorithm. We then start with the first neighborhood, keep solving until a specific stopping rule, and then use the resulting solution as an initial solution to be explored using the second neighborhood. We follow the VND as structured in [16] and in Algorithm 1 (see chapter I), except that we just move from the first neighborhood to the second one and solve until the stopping criteria of the second one. That is, we do not return back to the first one since we did not find any computational improvement in doing so and since the output of the second neighborhood is not feasible as an input to the first one. Note that for our algorithm, $k_{max} = 2$ since we only use our two complimentary neighborhoods.

We next discuss our experimental instances and show how our algorithms can be efficiently used to solve them.

4.5 Numerical Results

We apply our model to a food supply chain application. We start with describing the application. Then, we describe how we generated our realistic instances, and finally, show our numerical results.

The food supply chain that we consider is for one of the world's largest companies in the field of food processing and supplies. The company supplies numerous fast food restaurants world-wide with most of their goods. This includes liquid products, such as ketchup and mayonnaise, meat products, bakery, and others. The fast food restaurants (the customers), have a monthly demand for the different food products. The company manufactures a few of these products in its plants and outsources the rest. Even for the end products manufactured by the company, their raw materials are still purchased from outsider suppliers. All end products get stored in one of the multiple warehouses owned by the company before they get distributed to the restaurants to fulfill the demand. External suppliers for the company provide quantity discounts using the scheme earlier described in our model; the discount intervals are for the total annual orders, but the orders are placed each month.

Inspired by real data, we generated 30 numerical instances for our model. Locations for suppliers, facilities, and customer zones were generated on a grid of the US map, uniformly distributed among the main populous areas. Euclidean distances between these locations were calculated. All parameters were generated from uniform distributions between lower and upper bound values that are realistic with accordance to the data of the company. Table 5 shows the lower and upper bounds on the number of suppliers, facilities, customer zones, and raw materials that we used. All capacities, costs, and customer demand were generated in a similar way, with summer seasonal demand peaks put in consideration as well as an annual demand increases. Some of the raw materials are end products themselves. Other end products are processed by combining one or more raw materials at the company's facilities.

We assumed that suppliers offer six quantity discount intervals. The time horizon is exactly as described above in our model, i.e., there are five annual periods with the first one being further divided into twelve months.

We compare the performance of the commercial MIP solver CPLEX 12.6 with each of our three algorithms. Setting the MIP Emphasis parameter of CPLEX to 1 is supposed to make the solver focus on finding good feasible solutions quickly. However, we did not find any significant difference in the results of CPLEX whether we set that parameter to 1 or to its default value of 3 (which balances feasibility and optimality). We also tried tuning other parameters of CPLEX, but none of them had any significant difference on the results. Consequently, we report the results of using CPLEX with its default parameter values.

We warm-start all of our algorithms and CPLEX with our initial solution described above. Our experimentation showed that doing so yields the best results. Our initial solution turned out to be fairly good by itself; resulting in an average relative gap of approximately 18% among all instances. Without this warm-start, CPLEX does not get a solution with that quality gap within a three-hour time limit for the vast majority of our instances, and it does not even find any feasible solution for most of them. We computed Z_{best} by warm-starting CPLEX with the best solution found using all of our three algorithms and solving for 10 hours.

Table 5 Ranges of the Number of Suppliers, Facilities, Customer Zones, and Raw Materials for Our Numerical Instances

Parameter	**Minimum**	**Maximum**
Number of Suppliers	250	450
Number of Facilities	12	25
Number of Customer Zones	40	75
Number of Raw Materials	50	100

For the supplier selection LS, the sorting criterion that consistently worked the best was sorting the suppliers according to what we call the approximate total supplier purchasing cost. This cost is the sum of the fixed cost of selecting that supplier and the total purchasing cost if that supplier is the only selected one. This is calculated as follows: For supplier $i \in S$ that provides raw material $p \in RW$, we first calculate the total annual required quantity of product p by relating it through the BOM to all intermediate and final products that it is used in and multiplying that by the annual demand of these end products. We call this the $EquivalentDemand_{pt}$ for p in year $t \in T'$. Then, we multiply the result by the purchasing cost of the interval of that supplier that contains the $EquivalentDemand_{pt}$. If the $EquivalentDemand_{pt}$ is higher than the upper bound of the highest interval of the supplier, we calculate the equivalence of purchasing the maximum capacity of that supplier and add it to the penalty of not fulfilling the remaining $EquivalentDemand_{pt}$. We do this for all years and add up the total resulting cost. In mathematical form, this is:

$$Approximate\ total\ Purchasing\ Cost\ for\ Supplier\ i \in S\ That\ Provides\ Product\ p \in RM$$
$$= sc_{ip} + \sum_{t \in T'} \sum_{b \in B_{ip}} EquivalentDemand_{pt}.pc_{ipb}.I$$
$$+ \sum_{t \in T'} \Big[UB_{ip|B_{ip}|}.pc_{ip|B_{ip}|} + (EquivalentDemand_{ip} - UB_{ip|B_{ip}|}).Penalty\Big].I'$$

Where, $I = 1\ if\ LB_{ipb} \leq EquivalendDemand_{ip} \leq UB_{ipb}\ and\ 0\ otherwise,$ and $I' = 1\ if\ EquivalendDemand_{ip} > UB_{ip|B_{ip}|}\ and\ 0\ otherwise.$

For the raw material separation LS, we found that having equal random clusters works best compared to all the other clustering mechanisms that we described in subsection 4.4.3 above. We also found that fixing the continuous variables related to all raw materials not executed in any specific iteration speeds up the solution time significantly for that iteration without having a big effect on solution quality. We adopted this in our implementation. We used the following stopping criteria in each of the reduced MIPs of our algorithms: either getting a solution that is within 4% of the lower bound of that reduced MIP or reaching a 180 seconds limit.

Note that for our large realistic instances, we have no way to get the optimal solution and compare the solutions of our algorithms to it. For small instances, CPLEX can get the optimal solutions. We verified that by relaxing the aforementioned stopping criteria for the sub-problems of our second algorithm and let CPLEX solve each sub-problem to optimality since each sub-problem is a small instance of our model. CPLEX was able to get the optimal solution of each of these problems within 60 minutes. Since our algorithms are based on MIP-based LS, the underlying idea is that we use the solver to solve these sub-problems anyway. Thus, we note that for these smaller instances, our algorithms get a relative gap of 0.01% to the optimal solution (which is the default gap from optimal solution for CPLEX).

The instance generator, model generation, and the three algorithms were all coded in C++ where CPLEX 12.6 was called through Concert Technology [5]. All experiments were done on a Linux machine with Kernel 2.6.18 running a 64-bit x86 processor with two 2.27 GHz Intel Xeon E5520 chips and 32GB of RAM.

Figure 10 compares the average performance of CPLEX to each of our algorithms for a running time of one hour. CPLEX finds very few feasible solutions with minor improvements over our initial solution within the 1 hour limit. All three of our algorithms outperform CPLEX. The supplier selection LS is better than the raw material separation one until almost the 2000^{th} second, after which the latter gives better results. VND outperforms all other methods. It gets solutions within a 1% average relative gap in 33.18 minutes. This shows the benefit of combining both neighborhoods in the VND scheme.

We also ran all methods for two more hours to see if there is any difference in performance. Figure 11 plots the results of the four methods over a three-hour running time. There is a very slight improvement in CPLEX's results, but almost no improvement in any of our algorithms. However, all of our methods still well outperform CPLEX, and the comparison between them remains the same as the case of the one hour limit. It is worth noting that running CPLEX for even four more hours did not yield any significant improvement.

Table 6 shows the time at which the VND algorithm finds its best solution and the time each of the other 3 methods find that solution for all instances. A value of ">10800" for any method means that the method did not find a solution of similar or higher quality within the 3 hours/10800 seconds limit. None of the methods gets the best solution of the VND within the three hours limit for 21 out of the 30 instances. The supplier selection LS outperforms the VND in six instances, while the raw material separation LS outperforms it in three instances. CPLEX gets that solution faster than the VND algorithm in only one instance, gets it slower than the VND algorithm in one other instance, and never gets the solution within the three hours limit for the remaining 28 instances. These results emphasize how the leading commercial solver suffers in getting high quality feasible solutions quickly and also show the efficiency of our algorithms.

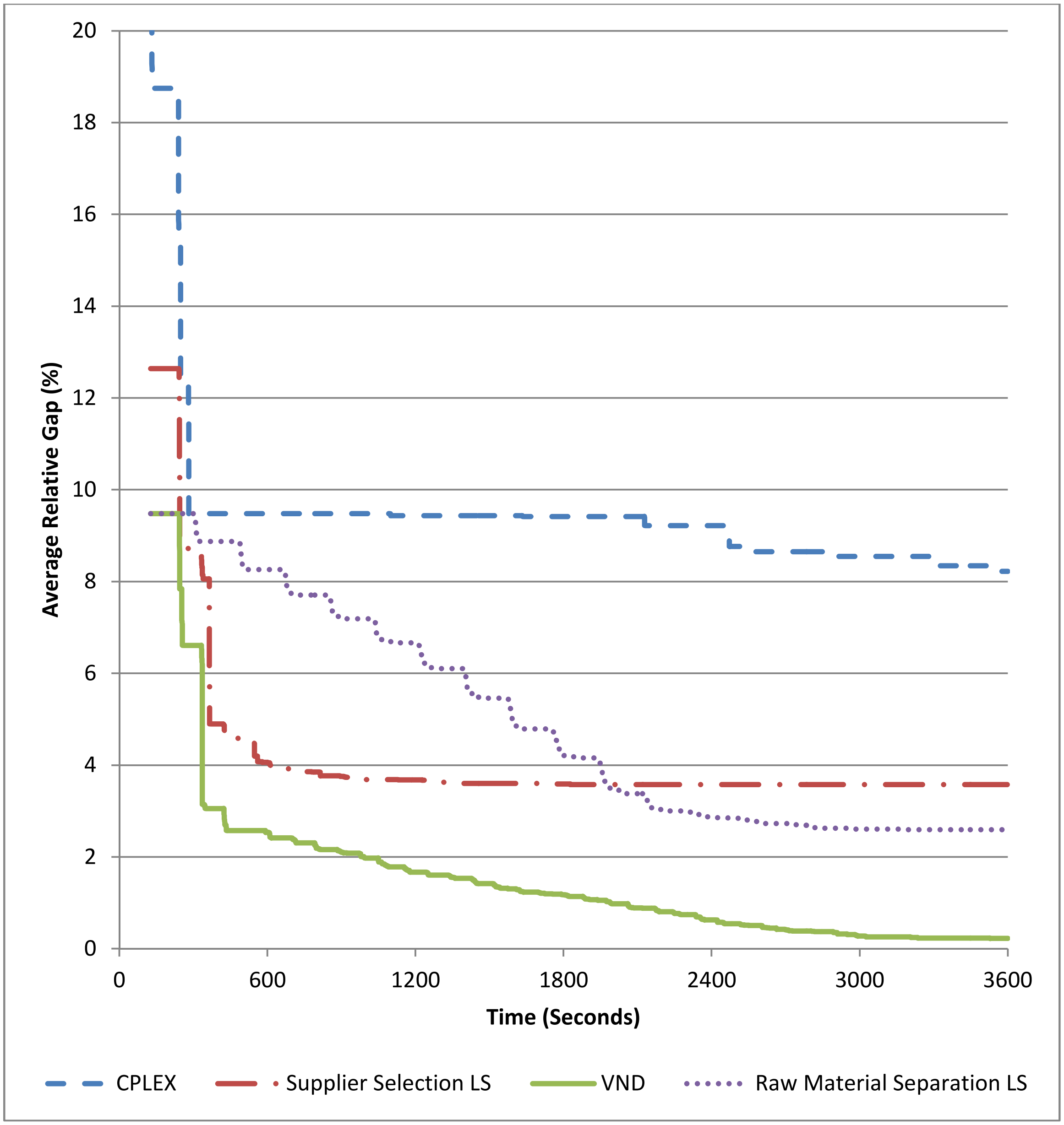

Figure 10 Comparisons between the Performance of Each of Our 3 Algorithms and CPLEX for an Hour

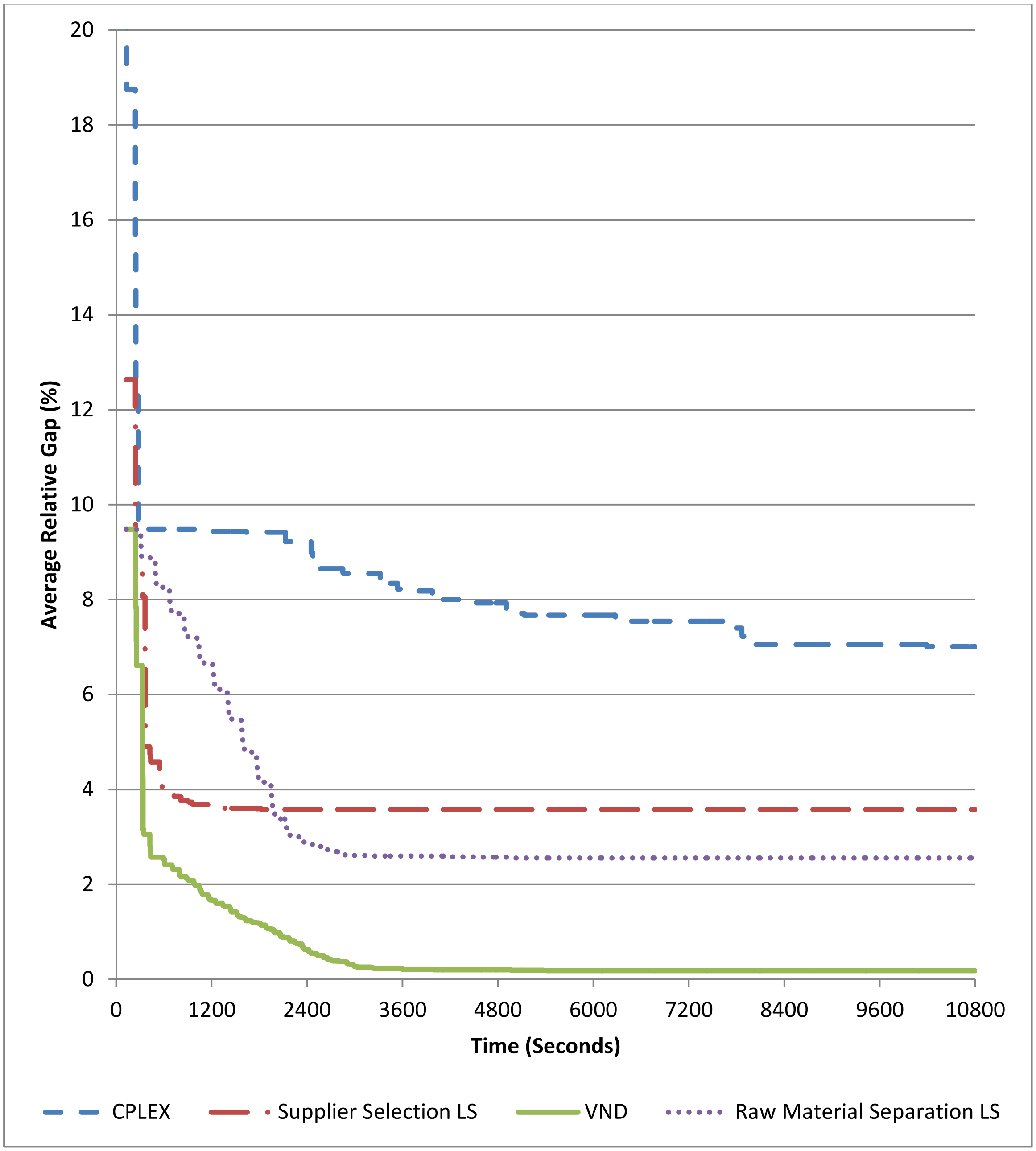

Figure 11 Comparisons between the Performance of Each of Our 3 Algorithms and CPLEX for 3 Hours of Computing Time

Table 6 Instance-by-Instance Comparison: Time of Best Solution for VND and Time Until Each of The Other Algorithms and CPLEX Reaches That Solution For Each Instance

Instance Number	Time of best solution of VND (Seconds)	Time until CPLEX gets best solution of VND (Seconds)	Time until supplier selection LS gets best solution of VND (Seconds)	Time until raw material separation LS gets best solution of VND (Seconds)
1	3004	> 10800	151	> 10800
2	5407	> 10800	152	1974
3	2518	> 10800	> 10800	> 10800
4	2433	> 10800	> 10800	> 10800
5	2983	> 10800	> 10800	> 10800
6	2550	7751	333	> 10800
7	2894	> 10800	> 10800	> 10800
8	2611	> 10800	> 10800	> 10800
9	2728	> 10800	> 10800	> 10800
10	5373	> 10800	152	> 10800
11	2564	> 10800	957	> 10800
12	2272	> 10800	> 10800	> 10800
13	3236	> 10800	> 10800	> 10800
14	2466	> 10800	611	> 10800
15	3091	> 10800	> 10800	2121
16	2354	> 10800	> 10800	> 10800
17	2703	> 10800	> 10800	> 10800
18	3174	4530	> 10800	> 10800
19	2632	> 10800	> 10800	> 10800
20	2950	> 10800	> 10800	> 10800
21	2605	> 10800	> 10800	2305
22	2338	> 10800	> 10800	> 10800
23	2992	> 10800	> 10800	> 10800
24	2697	> 10800	> 10800	> 10800
25	4598	> 10800	> 10800	> 10800
26	2614	> 10800	> 10800	> 10800
27	2551	> 10800	> 10800	> 10800
28	2366	> 10800	> 10800	> 10800
29	2357	> 10800	> 10800	> 10800
30	2348	>10800	>10800	>10800

Chapter V

CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

In this thesis, we presented modeling frameworks for three supply chain planning problems. We applied each of them to a real-world application. We present our conclusions and directions for future research in this last chapter.

5.1 Conclusions

In chapter II, we presented a real world supply chain planning application in the wind turbines industry, that has not been investigated before in the literature. We developed a comprehensive tactical mathematical model that is multi-commodity, multi-echelon, and dynamic. We applied it to our real-world case study. The model handles for the first time delay dependent backorder costs that can have any functional form.

We showed that the model enables efficient planning of tactical supply chain decisions in the wind turbines case study and similarly structured supply chains. For our case study the impact is a cost reduction from 15% to 85% if the backorder costs are considered explicitly in the model. In addition, we showed how being able to model backorder penalties with a piecewise convex cost structure resulted in a total cost reduction of up to 6% compared to using a linear cost approximation of the backorder penalties.

In chapter III, we developed a two-stage stochastic programming model for comprehensive tactical supply chain planning under uncertainty, inspired by a real-world application in the wind turbines industry. The model deals with multi-period, multi-product, and multi-echelon supply chains. Uncertainty/unreliability of suppliers in its most general form, which is a combination of random yield and stochastic lead times, is considered. We described a case study of our model in the wind turbines industry for one of the world's biggest manufacturers of this industry. In our experimental computational results that used realistic data from the wind turbines application, the model chooses the cheapest suppliers, regardless of their reliability, if feasible solutions using these cheap suppliers can be generated. They also showed that optimal solutions for the expected/mean value problem are higher than that of the corresponding case of completely reliable suppliers. In addition, the cost ratio of the MVP over the deterministic case increases with increasing backorder costs. The value of using a stochastic program over a deterministic one as a percentage of the optimal stochastic solution reached values of up to 20% for the experiments we carried out. We also showed and analyzed that the optimal procurement quantities might be higher than the demand in some cases.

In chapter IV, we presented a mixed integer programming model for a production-distribution planning problem that incorporates a novel time-aggregated quantity discounts scheme. The model is very hard to solve using leading commercial solvers. With the aim of getting good feasible solutions quickly, we developed an algorithm that constructs a good initial solution and three other iterative algorithms that are capable of very quickly getting high quality primal solutions. Two of the latter three algorithms are based on MIP-based local search and one is a VND combination the two. We presented numerical results from a realistic food supply chain and showed the efficiency of our customized algorithms. The leading commercial solver CPLEX finds very few if any feasible solutions with minor improvements over our initial solution within a three hours solution time limit. All our algorithms well outperform CPLEX. The VND algorithm has the best average performance. Its average relative gap to the best known feasible solution is within 1% in less than 35 minutes of computing time.

5.2 Directions for Future Research

A number of interesting directions for future research exist. For the first problem, adding more global supply chain considerations is one direction for future research; since the wind turbine supply chain under study is a global one. Examples of these considerations include taxes, tariffs, transfer prices regulations, and currency exchange rates. In addition, adding supplier selection issues to the same problem is another extension. Note that our model in chapter IV includes supplier selection but it does not include the possibility of backordering. Furthermore, some countries enforce a minimum percentage of local content of the raw materials used to manufacture end products sold in these countries. That adds an extra restriction on the chosen suppliers and procurement plans. One can add this restriction and study its effect on the overall supply chain costs.

Our model in chapter III deals with tactical planning, and so suppliers were assumed to have been selected. One extension would be adding the issue of supplier selection with their fixed contractual costs. The resulting model would have integral variables, and might be challenging to solve. The use of a primal decomposition method such as Benders decomposition may be warranted. Also, adding more global supply chain issues, e.g., customs, tariffs, local content, taxes, etc., is another extension of this work. A third extension of this work is to apply it to other relevant real-world applications

For our work in chapter IV, one direction for extending this work is to include uncertainty and use a stochastic programming approach to model the problem. In this case, one can use our algorithms to efficiently solve the one-scenario problem of the stochastic models; as these sub-problems would be nothing but the deterministic model discussed in chapter IV. Note that the stochastic model in chapter III did not include supplier selection or quantity discounts. Another direction for future research, as indicated for the previous two problems, would be including the international aspects of supply chain planning. Then, one can study the impact of these international considerations on the overall supply chain costs.

APPENDIX A

PROOF OF COROLLARY 1

Corollary 1:

A linear backorder cost structure with a positive intercept always satisfies condition 2.27.

Proof:

Let $a > 0$ be the intercept of that linear cost structure function, and *s* be its slope. |*T*| is the cardinality of the set of periods *T* (so the biggest period difference is |*T*|-*1*). Figure 12 shows this cost structure.

Now, for any three period differences $\Delta_1, \Delta_2,$ and Δ_3, where $0 < \Delta_1 \le \Delta_2 < \Delta_3 \le T$ and $\Delta_1 + \Delta_2 = \Delta_3$. Since $a > 0,$ then $a + s\Delta_3 < 2a + s\Delta_3$. Also, since $\Delta_1 + \Delta_2 = \Delta_3$, then we get that $a + s\Delta_3 < a + s\Delta_1 + a + s\Delta_2$. This last identity implies that it will always be cheaper to have a backorder for a difference in periods equal to Δ_3 in one step/jump, rather than doing it in two jumps (by having a backorder flow for a difference in periods of Δ_1 and another one of Δ_2). This prevents the two-hop solution from becoming the optimal solution. Recursively, this also holds for any number of possible jumps, and thus satisfies condition 2.27. ■

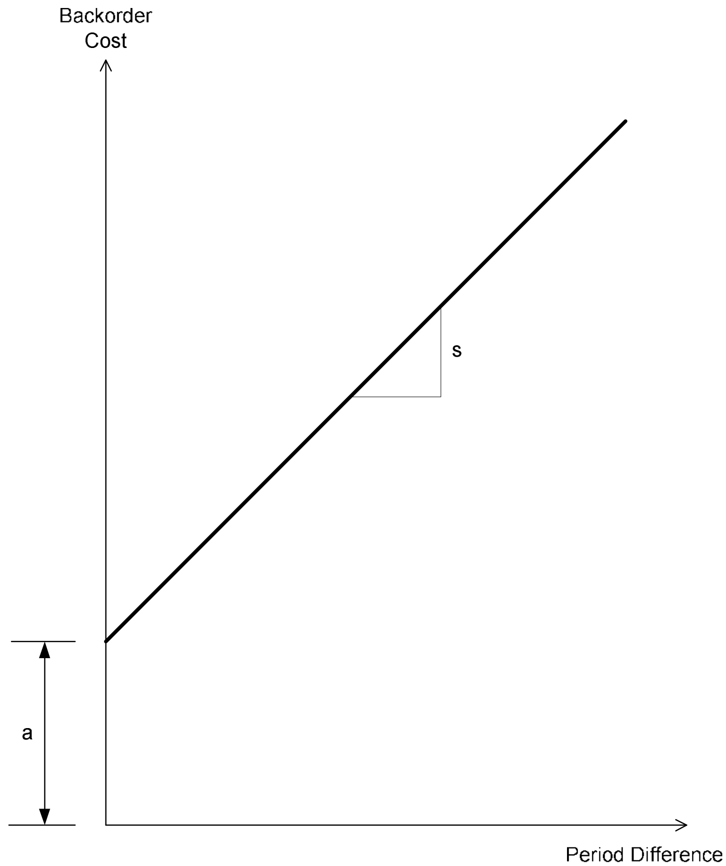

Figure 12 A Linear Backorder Cost Structure

APPENDIX B

PROOF OF COROLLARY 2

<u>*Corollary 2:*</u>

A piecewise linear concave backorder cost structure, with a positive intercept of the first interval, always satisfies condition 2.27.

<u>*Proof:*</u>

We first prove it for the case of a curve with two intervals and then extend it to the case of more than two intervals. For the case of two intervals, let $a_1 > 0$ and $a_2 > 0$ be the intercepts and let s_1 and s_2 be the slopes of the first and second linear segments, respectively, $\Delta_{1,2}$ be the period difference at which the slope changes from s_1 to s_2, and $|T|$ be the cardinality of the set of periods *T*. Figure 13 illustrates this cost structure function. Since the cost curve is piecewise linear concave, then $a_1 < a_2$, and $s_1 > s_2 > 0$.

For any three period differences Δ_1, Δ_2, and Δ_3, where $0 < \Delta_1 \leq \Delta_2 < \Delta_3 \leq |T|$ (assuming without loss of generality that $\Delta_1 \leq \Delta_2$), and $\Delta_1 + \Delta_2 = \Delta_3$, there are only four possible cases:

Case 1: $\Delta_3 \leq \Delta_{1,2}$, that is all period differences belong to the first linear segment of the backorder cost structure. In this case, according to proposition 1, condition 2.27 is satisfied.

Case 2: $\Delta_1 \geq \Delta_{1,2}$. Here all period differences belong to the second linear segment. Similarly, using proposition 1, the condition is satisfied.

Case 3: $\Delta_1 < \Delta_{1,2}$ and $\Delta_2 > \Delta_{1,2}$ (i.e., Δ_1 belongs to the first segment and both Δ_2 and Δ_3 belong to the second segment). $s_1 > s_2 \Rightarrow \Delta_1(s_2 - s_1) < 0$ as $\Delta_1 > 0$. Since $a_1 > 0$, then $\Delta_1(s_2 - s_1) < a_1 \Rightarrow a_2 + s_2\Delta_1 < a_2 + a_1 + s_1\Delta_1$.

Using $\Delta_1 + \Delta_2 = \Delta_3$, we get that $a_2 + s_2\Delta_3 < a_1 + s_1\Delta_1 + a_2 + s_2\Delta_2$, which satisfies the condition preventing backordered flows that make two hops from being the optimal solution. Recursively, the same applies to the prevention of having more than two jumps in the optimal solution.

Case 4: $\Delta_2 \leq \Delta_{1,2}$ and $\Delta_3 > \Delta_{1,2}$ (i.e., both Δ_1 and Δ_2 belong to the first segment, and Δ_3 belongs to the second segment). Using these inequalities, that $s_1 > s_2 > 0$, and that $\Delta_{1,2} > 0$, we get that $(s_1 - s_2)(\Delta_3 - \Delta_{1,2}) > 0$.

Since $a_1 > 0$, then $-a_1 < (s_1 - s_2)(\Delta_3 - \Delta_{1,2}) \Rightarrow -a_1 < \Delta_3(s_1 - s_2) - \Delta_{1,2}(s_1 - s_2) \Rightarrow$ $-a_1 + s_1\Delta_{1,2} - s_2\Delta_{1,2} + s_2\Delta_3 < s_1\Delta_3$. Noticing that $\Delta_{1,2}$ lies on both lines, we get that $a_1 + s_1\Delta_{1,2} = a_2 + s_2\Delta_{1,2}$ (i.e., $s_1\Delta_{1,2} - s_2\Delta_{1,2} = a_2 - a_1$). Also, $\Delta_3 = \Delta_1 + \Delta_2$. Substituting those last two equalities, we get that $a_2 + s_2\Delta_3 < a_1 + s_1\Delta_1 + a_1 + s_1\Delta_2$, which, again, satisfies the condition for the case of two hops. Recursively, the same applies to the prevention of having more than two hops. So, we conclude that the condition is satisfied in all cases.

We now prove the above proposition for the case of a linear cost structure with more than two segments. We here define a_i and s_i as the intercept and slope of line segment i, respectively. We also define each $\Delta_{i-1,i}$ as the period difference at which the slope increases from s_{i-1} for piece $i-1$ to s_i for the successive segment i. Note that $\Delta_{i-1,i}$ is the intersection of the line with intercept a_{i-1} and slope s_{i-1}, and the line with intercept a_i and slope s_i. More generally, $\Delta_{j,i}$ is the intersection of the line with intercept a_j and slope s_j, and the line with intercept a_i and slope s_i, where $a_i > a_j, s_i < s_j$, and both segments belong to the piecewise linear concave cost structure, but they are not successive. From concavity, for any $1 \le j \le i-1$, $\Delta_{i-1,i} \ge \Delta_{j,i}$ (see figure 14). We have four cases here:

Case 1: Δ_1, Δ_2, and Δ_3 belong to the same segment. In this case, from proposition 1, condition 2.27 is satisfied.

Case 2: Δ_1, Δ_2 belong to the same segment j, and Δ_3 belongs to the segment $i \ge j$. Since, $\Delta_3 \ge \Delta_{i-1,i}$ and $\Delta_{i-1,i} \ge \Delta_{j,i}$, then $\Delta_3 \ge \Delta_{j,i}$. Then similarly to the proof of case 4 in the linear piecewise concave cost structure with 2 linear segments this case satisfies condition 2.27.

Case 3: Δ_1 belongs to the linear segment j, and both Δ_2 and Δ_3 belong to the segment $i > j$. Here, since, $\Delta_3 \ge \Delta_{i-1,i}$ and $\Delta_{i-1,i} \ge \Delta_{j,i}$ (from concavity), then $\Delta_3 \ge \Delta_{j,i}$. From the previous two proofs, this case also satisfies the condition 2.27.

Case 4: Δ_1 belongs to the linear piece j, Δ_2 belongs to the linear piece $k > j$, and Δ_3 belongs to the piece $i > k$. Since $\Delta_1 + \Delta_2 = \Delta_3$, $\Delta_{k.i} = \dfrac{a_i - a_k}{s_k - s_i}$ (because

$a_k + s_k\Delta_{k,i} = a_i + s_i\Delta_{k,i}$ as $\Delta_{k,i}$ belongs to both lines k and i), and $\Delta_3 \geq \Delta_{k,i}$ (as proved in case 1), we get:

$\Delta_1 + \Delta_2 \geq \dfrac{a_i - a_k}{s_k - s_i} \Rightarrow a_k + (s_k - s_i)\Delta_1 + (s_k - s_i)\Delta_2 \geq a_i$. Now, since $(s_j - s_i) > (s_k - s_i)$ (from concavity), and $a_j > 0$ (since $a_j \geq a_1$, where a_1 is the positive intercept of the first linear piece as given, then $a_j + (s_j - s_i)\Delta_1 + a_k + (s_k - s_i)\Delta_2 > a_i$. Using $\Delta_1 + \Delta_2 = \Delta_3$, we get $a_j + s_j\Delta_1 + a_k + s_k\Delta_2 \geq a_i + s_i\Delta_3$. Then, for all three cases, using the same argument we used at the end of our proof of proposition 1, condition 2.27 is satisfied. ■

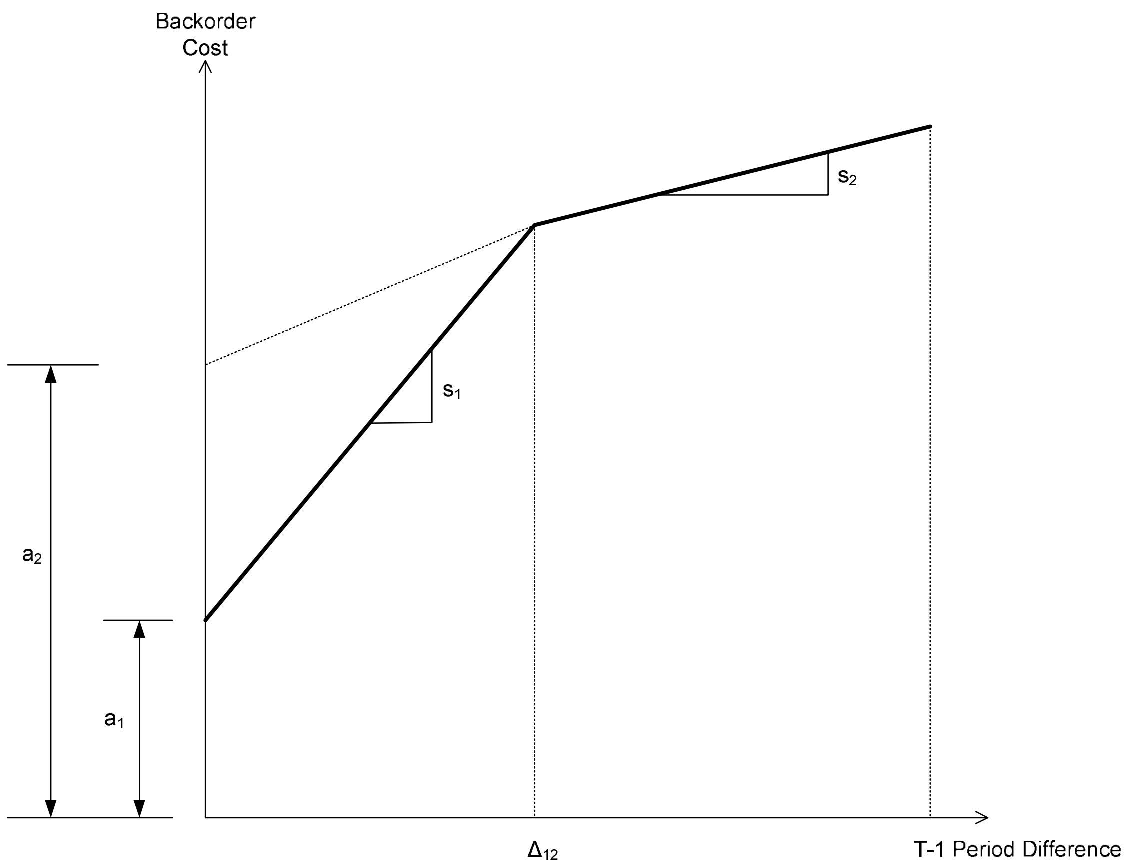

Figure 13 Piecewise Linear Concave Backorder Cost Structure with Two Pieces

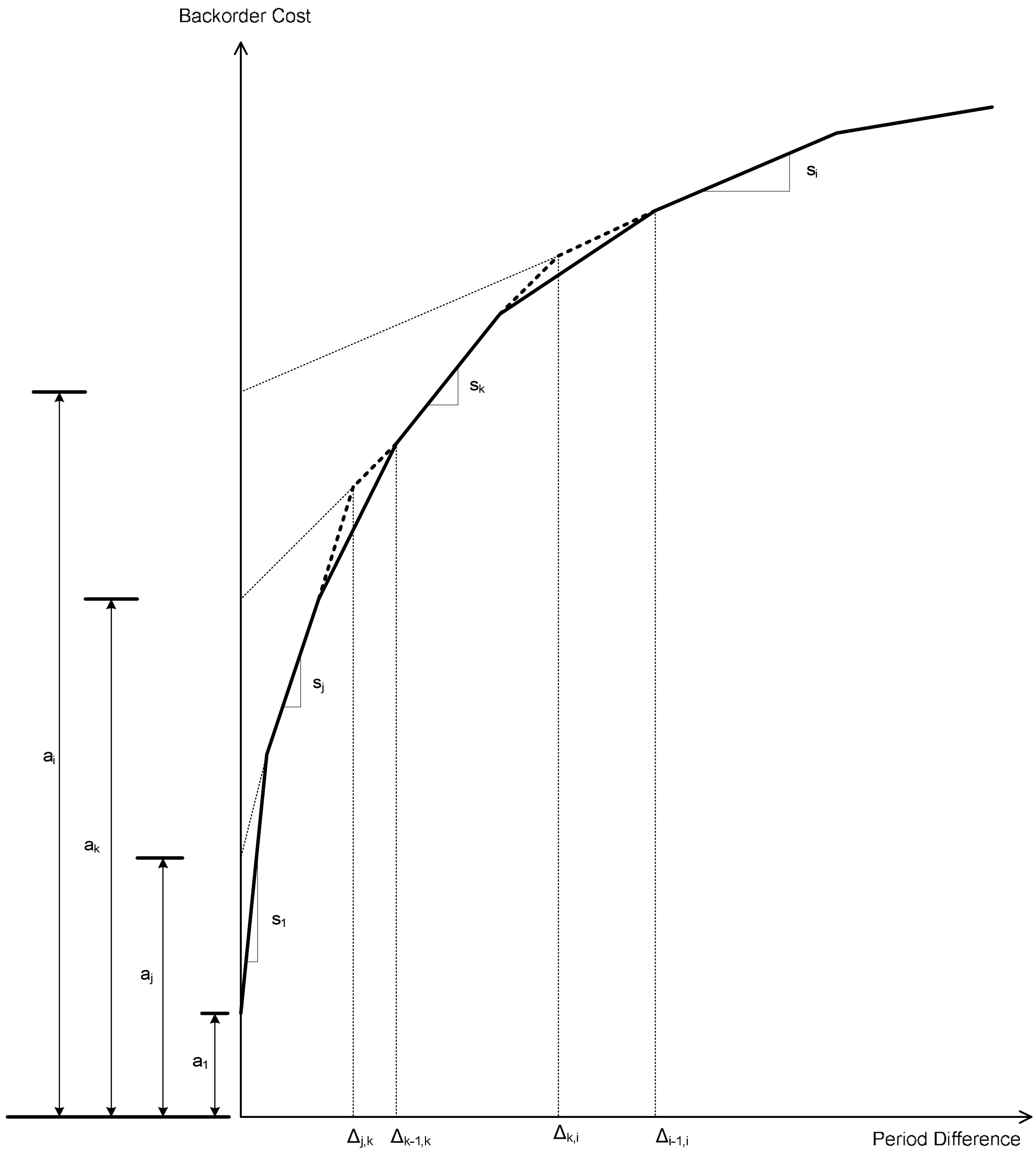

Figure 14 Piecewise Linear Concave Backorder Cost Structure with More Than Two Segments

APPENDIX C

SUMMARY OF THE NOTATION USED IN SECTION 3.3

Below is a summary of the notation used in our models in Section 3.3 and appendix D.

Sets:

S	Set of suppliers
TF	Set of transformation facilities
P	Set of products
T	Set of periods
R	Set of resources
K	Set of customers
FR, AR, IR, TR	Sets of resources required for product flow (FR), assembly (AR) and product inventory (IR) in transformation facilities, and those required for product transportation between origins and destinations (TR), and those required for production at suppliers (SR). These are the subsets of the set of resources R
D	Set of destinations, $D = TF \cup K$
Ω	Set of scenarios

Parameters:

pc_{ipt}	Purchasing cost per unit of product $p \in P$ from supplier $i \in S$, when ordered in period $t \in T$

Max_{ipt}	Maximum possible order quantity of product $p \in P$ from supplier $i \in S$ in period $t \in T$
tc_{ijpt}	Transportation cost per unit of product $p \in P$ shipped from supplier $i \in S$ and transformation facility $j \in TF$, or from transformation facility $i \in TF$ and destination $j \in D$ in period $t \in T$
$fc_{jpt,}$ ac_{jpt}, ic_{jpt}	Throughput cost, assembly cost, and inventory cost, respectively, per unit of product $p \in P$ at transformation facility $i \in TF$ in period $t \in T$
pn_{kpt}	Penalty per unit of lost sales of product $p \in P$ for customer $k \in K$ in period $t \in T$
bc_{kptu}	Backorder cost per unit of product $p \in P$ when it is delivered to customer $k \in K$ in period $t \in T$ to satisfy part of or the whole demand of that product at that customer for the earlier period $u \in \{1, \cdots, \|T\|\}$, where $\|T\|$ is the cardinality of the set of periods T
$ccapexp_{ijpt}$	Cost of expanding the transportation capacity per unit of product $p \in P$ shipped from supplier $i \in S$ to transformation facility $j \in TF$, or from transformation facility $i \in TF$ to destination $j \in D$ in period $t \in T$
$cfcapexp_{jpt}$, $cacapexp_{jpt}$, $cicapexp_{jpt}$	Cost of expanding the throughput, assembly/manufacturing, and inventory capacity, respectively, per unit of product $p \in P$ at transformation facility $j \in TF$ in period $t \in T$
crc_{ijrt}	Resource cost per unit of transportation resource $r \in TR$ shipped from supplier $i \in S$ to transformation facility $j \in TF$, or from transformation facility $i \in TF$ to destination $j \in D$ in period $t \in T$

frc_{jrt}, arc_{jrt}, irc_{jrt}	Unit resource cost of resource $r \in TR$ for flow, $r \in AR$ for assembly (production), and $r \in IR$ for inventory, respectively, at transformation facility $j \in TF$ in period $t \in T$
$cres_{ijprt}$	Quantity of transportation resource $r \in TR$ required to ship one unit of product $p \in P$ from supplier $i \in S$ to transformation facility $j \in TF$, or from transformation facility $i \in TF$ to destination $j \in D$ in period $t \in T$
$fres_{jprt}$, $ares_{jprt}$ $ires_{jprt}$,	Quantity of flow resource $r \in FR$, assembly/manufacturing resource $r \in AR$, and inventory resource $r \in IR$, respectively, required per unit of product $p \in P$ at transformation facility $j \in TF$ in period $t \in T$
$rccapexp_{ijrt}$	Cost of expansion of one unit of transportation resource $r \in TR$ used for shipment from supplier $i \in S$ to transformation facility $j \in TF$, or from transformation facility $i \in TF$ to destination $j \in D$ in period $t \in T$
$rcfcapexp_{jrt,}$ $rcacapexp_{jrt,}$ $rcicapexp_{jrt}$	Cost of expansion of one unit of flow resource $r \in FR$, assembly/manufacturing resource $r \in AR$, and inventory resource $r \in IR$, respectively, at transformation facility $j \in TF$ in period $t \in T$
$ps_{iptt'}$	Penalty per unit of product $p \in$ P paid by supplier $i \in$ S for supplying orders placed for period $t' \in$ T, at period $t \in \{t', \cdots, \ |T|\}$, where $|T|$ is the cardinality of the set of periods T
$ccap_{ijpt}$	Capacity of transporting product $p \in P$ from supplier $i \in S$ to transformation facility $j \in TF$, or between transformation facility $i \in TF$ to destination $j \in D$ in period $t \in T$

$fcap_{jpt}$, $acap_{jpt}$, $icap_{jpt}$	Flow, assembly/manufacturing, and inventory capacity, respectively, for product $p \in P$ in period $t \in T$
$ccap_{ijrt}$	Capacity of transportation resource $r \in TR$ used for shipment from supplier $i \in S$ to transformation facility $j \in TF$, or from transformation facility $i \in TF$ to destination $j \in D$ in period $t \in T$
$fcap_{jrt}$, $acap_{jrt}$, $icap_{jrt}$	Capacity of flow resource $r \in FR$, assembly/manufacturing resource $r \in AR$, and inventory resource $r \in IR$, respectively, at transformation facility $j \in TF$ in period $t \in T$
$1bom_{pv}$	Number of units of component $p \in P$ required to assemble one unit of assembly (semi-finished or finished product) $v \in P$ in period $t \in T$ at transformation facility $j \in TF$ where component p is an element of the single level bill of material of product v
re_{jpt}	Received quantity of product $p \in$ P at transformation facility $j \in$ TF in period $t \in$ T, that was ordered in the previous planning horizon, but will arrive in period $t \in$ T of the current planning horizon
$init\_inv_{jp}$	Initial inventory or product $p \in$ P at transformation facility $j \in$ TF
$\Delta_{iptt'}$	Percentage of order quantity of product $p \in$ P ordered from supplier $i \in$ S in period $t' \in$ T that will be delivered in period $t \in \{t', \cdots, \|T\|\}$, where $\|T\|$ is the cardinality of the set of periods T (only in deterministic model)
dem_{kpt}	Demand of product $p \in P$ at customer $k \in K$ in period $t \in T$ (only in

the deterministic model)

$\Delta_{iptt'}(\omega)$ Percentage of order quantity of product $p \in$ P ordered from supplier $i \in$ S in period $t' \in$ T that will be delivered in period $t \in \{t', \cdots, |T|\}$, where $|T|$ is the cardinality of the set of periods T for scenario $\omega \in \Omega$ (only in the stochastic model)

$dem_{kpt}(\omega)$ Demand of product $p \in P$ at customer $k \in K$ in period $t \in T$ for scenario $\omega \in \Omega$ (only in the stochastic model)

Decision Variables:

pq_{ijpt} Purchased quantity of product $p \in$ P from supplier $i \in$ S to transformation facility $j \in$ TF, that is supposed to arrive in period $t \in$ T (part or the whole of it might be delayed and delivered in later periods; due to supplier random unreliability.

$x_{ijpt}(\omega)$ Delivered quantity of product $p \in$ P from supplier $i \in$ S to transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

$y_{ijpt}(\omega)$ Delivered quantity of product $p \in$ P from transformation facility $i \in$ TF to destination $j \in$ TF $\cup K$ in period $t \in$ T for scenario $\omega \in \Omega$

$iq_{jpt}(\omega)$ Inventory quantity of product $p \in$ P held at the end of period $t \in$ T at transformation facility $j \in$ TF for scenario $\omega \in \Omega$

$cq_{jpvt}(\omega)$ Quantity of product $p \in$ P used to assemble/manufacture product $v \in$ P at transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

$aq_{jpt}(\omega)$ Quantity of product $p \in$ P assembled/manufactured at transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

$ls_{kpt}(\omega)$ Lost sales quantity of product $p \in P$ at customer $k \in K$ in period $t \in T$ for scenario $\omega \in \Omega$

$bq_{kptu}(\omega)$ Backorder quantity of product $p \in$ P shipped to customer $k \in$ K in period $t \in$ T to fulfill part of the whole demand of that product at that customer for period $u \in \{1, \cdots, \mathrm{t} - 1\}$, for scenario $\omega \in \Omega$

$ccExp_{ijpt}(\omega)$ Quantity of transportation capacity expansion for product $p \in$ P, transported from origin $i \in$ S ∪ TF to destination $j \in$ TF ∪ K in period $t \in$ T for scenario $\omega \in \Omega$

$fcapExp_{jpt}(\omega)$ Quantity of throughput capacity expansion for product $p \in$ P at transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

$icapExp_{jpt}(\omega)$ Quantity of inventory capacity expansion for product $p \in$ P at transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

$acapExp_{jpt}(\omega)$ Quantity of assembly/manufacturing capacity expansion for product $p \in$ P at transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

$RccapExp_{ijrt}(\omega)$ Quantity of transportation resource capacity expansion for resource $r \in$ TR, transported from origin $i \in$ S ∪ TF to destination $j \in$ TF ∪ K in period $t \in$ T for scenario $\omega \in \Omega$

$RfcapExp_{jrt}(\omega)$ Quantity of throughput resource capacity expansion for resource $r \in$ FR at transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

$RicapExp_{jrt}(\omega)$ Quantity of inventory resource capacity expansion for resource $r \in$ IR at transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

$RacapExp_{jrt}(\omega)$ Quantity of assembly/manufacturing resource capacity expansion for resource $r \in$ AR at transformation facility $j \in$ TF in period $t \in$ T for scenario $\omega \in \Omega$

APPENDIX D

SUFFICIENT CONDITION THAT PREVENTS THE ISSUE DISCUSSED IN SECTION 2.4.2 FROM HAPPENING

We give a sufficient condition that prevents the behavior/issue described in Section 2.4.2 from happening for the simple case of having no BOM. We assume that it is cheaper to store any unit of any product in any specific transformation facility at any specific time period than to both transport it and store it in another facility. Thus, $ic_{jpt} + \sum_{r\in R} irc_{jrt}.ires_{jprt} \leq tc_{jipt} + ic_{ipt} + \sum_{r\in R} irc_{irt}.ires_{iprt} \quad \forall j \in TF, \forall i \in TF\backslash\{j\}, \forall p \in P, \forall t \in T$. In addition, we assume that the backorder cost of any product $v \in End$ is a non-decreasing function of the backorder delay. We also assume that the total ordering quantity is equal to the total demand of end products, if all suppliers were completely reliable. Note that for that last assumption to hold, only the lost sales cost has to be sufficiently high, i.e., higher than the costs of ordering, transforming, and handling products.

We will present our results for the expected/mean value problem, and then we show how to generalize it to the stochastic problem. Note also that, for this former problem, suppliers are still unreliable; as they still have a reliability index of $\Delta_{iptt'} = \mathbb{E}_\omega\left[\Delta_{iptt'}(\omega)\right] \quad \forall i \in S, \forall p \in P, \forall t' \in T, \forall t \in \{t', \dots, T\}$, but with just one possible scenario for that unreliability. We use the convention that $\sum_{\{i,\dots,j\}}. = 0$ for $i > j$, and define $|.|$ of a set $.$ as the cardinality of the set.

We next present the aforementioned sufficient condition for the described case in theorem 3. The intuition is the following: since the issue of ordering extra quantities happens to avoid potential extra capacity, resource utilization, and backordering costs, if we guarantee that these costs are cheaper than purchasing those extra quantities and eventually storing them, this issue will never happen.

<u>*Theorem 3:*</u>

Under the aforementioned assumptions, the following condition is sufficient to guarantee that the total ordering quantity for each raw material in the case of the expected/mean value problem does not exceed that of the case of completely reliable suppliers by more than $\delta.|S|.|T|^2.|TF|.|C|$ *units* ($\delta \geq 0$) *for the case of no BOM:*

$$\forall i \in S, \forall p \in P, \forall t \in T, \forall t' \in T, \forall l \in T, \forall j \in TF, \forall k \in C:$$

$$\sum_{t\in\{l+1,\dots,End_{ipt'}\}} \Bigg(bc_{kptl} + fc_{jpt} + cfcapexp_{jpt} - \min_{c\in\{t',\dots,End_{ipt'}\}} fc_{jpc} + \sum_{r\in R} fres_{jprt}.\left(frc_{jrt} + rcfcapexp_{jrt}\right) - \sum_{r\in R} \min_{c\in\{t',\dots End_{ipt'}\}} fres_{jprc}.frc_{jrc} + tc_{jkpt} + ccapexp_{jkpt} - \min_{c\in\{t',\dots,End_{ipt'}\}} tc_{jkpc} + \sum_{r\in R} cres_{jkprt}.\left(crc_{jkrt} + rccapexp_{jkrt}\right) - \sum_{r\in R} \min_{c\in\{t',\dots End_{ipt'}\}} cres_{jkprc}.crc_{jkrc} \Bigg).\Delta_{iptt'}.dem_{kpl}$$

$$< pc_{ipt'}.\delta + \sum_{t\in\{t',\dots,End_{ipt'}\}} \left(tc_{ijpt} + \sum_{r\in R} crc_{ijrt}.cres_{ijprt}\right).\Delta_{iptt'}.\delta$$

$$+ \sum_{t\in\{t',\dots,l-1\}} \left(ic_{jpt} + \sum_{r\in R} irc_{jrt}.ires_{jprt}\right).\left(\sum_{c\in\{t',\dots,t\}} \Delta_{ipct'}\right).\delta$$

$$+ \sum_{t\in\{End_{ipt'}+1,\dots,|T_{new}|\}} \left(ic_{jpt} + \sum_{r\in R} irc_{jrt}.ires_{jprt}\right).\delta$$

$$+ \sum_{t\in\{l,\dots,End_{ipt'}\}} min\left\{\left(ic_{jpt} + \sum_{r\in R} irc_{jrt}.ires_{jprt}\right), bc_{kptl}\right\}.\Delta_{iptt'}.\delta$$

$$- \sum_{t\in\{t'+1,\dots,End_{ipt'}\}} ps_{iptt'}.\Delta_{iptt'}.\delta$$

Proof:

We argue by contradiction. Suppose that the optimal solution has a total ordering quantity of a specific product larger than the total demand for that product, which is equal to the total ordering quantity for the case of completely reliable suppliers by more than $\delta.|S|.|T|^2.|TF|.|C|$, while the above condition holds. This implies that ordering, from one supplier $i \in S$ (or more) in one of the periods $t \in T$ (or more) shipped to one of the transformation facilities $j \in TF$, is δ units higher than what it should be if the total ordering had been similar to the case of completely reliable suppliers. We now construct the following solution: order δ units less from each of these suppliers. Since the only benefit of ordering more is to try to fulfill the demand earlier, the solution that we are constructing might incur some additional costs as follows:

- An additional cost of backordering part of the demand from some of the periods succeeding the period of that demand.

- The costs of the potential need to expand the throughput and the throughput resource capacity of the facility in the periods that succeed that period of the demand; in order to assure the feasibility of the solution we are constructing. That is because in the optimal solution we assumed, there might be less (or no) throughput in the periods succeeding the demand period. Hence, we do not guarantee that there is enough throughput capacity for these periods.
- Similar costs to the two previous ones, but for the transportation capacity and transportation resource capacity
- Those backordered quantities are transported in later periods than the case of ordering extra quantities. Since the throughput and transportation costs, in addition to throughput and transportation resource usage, might be more expensive in those later periods; this potential increase in costs needs to be included here. We add an upper bound for these costs, which is adding them at those later periods, and subtracting the corresponding cheapest costs among all periods.

Everything else would remain feasible in the solution we are constructing; since it is the exact same as the optimal solution, but in lesser quantities. Therefore, all capacity and resource utilizations will be no larger than those of that optimal solution. An upper bound on these total additional costs can be constructed by calculating these costs for the highest case of ordering the total demand of that product, and needing to expand the throughput capacity and resources for the total amount of delivered quantities in each period higher than the demand period. The L.H.S. of the above condition is exactly this upper bound.

On the other hand, the optimal solution has at least the following additional costs compared to the solution we are constructing:

- The cost of purchasing the extra δ units.
- The cost of transporting the extra units.
- The cost of transportation resource utilization for the extra units.

Note that each of the last three costs is not paid at once as the first cost of purchasing the extra units, but rather in batches during the periods starting from when the order is placed until the last period in which part of the order will be received (depending on the reliability index of the supplier).

- The cost of holding extra δ units in inventory for all periods starting from the period when the demand is placed until right before the period of the demand.
- The cost of holding the extra δ units in inventory for all periods starting from the period right after the last period in which part of the order will arrive ($End_{ipt'}$) until the last period of the planning horizon (after the potential addition of some periods as described above). There are only two other alternatives to storing those extra units in these periods. One is to ship them to another transformation facility and store them there. However, we assumed that such alternative is more expensive than just storing them in the transformation facility they were shipped to from the suppliers. The other alternative is backordering part of the demand in these periods instead of fulfilling it earlier (either on time or by backordering from an earlier period). Nevertheless, that would still not happen; because of our assumption on the non-decreasing backorder cost as a function of the backorder delay.

- The cost of storage resource utilization for the extra units, for the same periods as the previous two storage costs. These periods do not include the periods between the actual period of the demand until the period $End_{ipt'}$; since during these periods, extra units that are received might get backordered. That adds the next extra cost:
- The minimum of either backordering or storing the extra received δ units during the periods between the actual demand period and the period $End_{ipt'}$.

In addition, there is an additional negative cost corresponding to the extra penalty paid by the supplier for delaying the extra δ units, according to its reliability index.

The R.H.S. of the above condition captures these costs giving a lower bound on the additional cost incurred by extra ordering.

Note that in the above condition, t' is the period in which the order is placed, and l is the demand period. Also, note that the above condition captures every $t' \in T$ and $l \in T$. It is also for every transformation facility, each supplier, product, and customer. Finally, note that the costs of the upper bound R.H.S. are separable among all the aforementioned indices.

Now, since the L.H.S. in the above condition is strictly less than its R.H.S., then the total cost of the solution we constructed is strictly less than that of the optimal solution, which is a contraction. ■

If the condition holds for each scenario $\omega \in \Omega$ of the stochastic version of the problem, then clearly, its optimal solution will not have any extra purchased quantities ordering.

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